

**Engineering  
IIT-JEE**



**Medical  
PMT/NEET**

*Where are you going? Where do you want to be?  
inspiring your future*

Where is  
my career?



**APEX EDUCATION**  
IIT-JEE / PMT / FOUNDATION / CBSE  
*Success through hard work & disciplined study ...*

## PHYSICS

### MOTION IN A STRAIGHT LINE

- The area under the velocity-time curve and time axis gives the displacement of the object for given interval of time.
- If a body falls freely, the distance covered by it in each subsequent second starting from first second will be in the ratio 1 : 3 : 5 : 7, etc.
- If a body is thrown vertically up with an initial velocity  $u$ , it takes  $u/g$  second to reach maximum height and  $u/g$  second to return, if air resistance is negligible.
- If air resistance acting on a body is considered, the time taken by the body to reach maximum height is less than the time to fall back the same height.
- For a particle having zero initial velocity if  $s \propto t^\alpha$ , where  $\alpha > 2$ , then particle's acceleration increases with time.
- For particle having zero initial velocity if  $s \propto t^\alpha$ , where  $\alpha < 0$ , then particle's acceleration decreases with time.
- Kinematic equations:  

$$v = u + at; v^2 = u^2 + 2as, \quad s = ut + \frac{1}{2}at^2$$

applicable only when particles moves with constant acceleration
- If acceleration is variable use calculus approach.
- Relative velocity  $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

### MOTION IN A PLANE

- If  $T$  is the time of flight,  $h$  maximum height,  $R$  horizontal range of a projectile,  $\alpha$  its angle of projection, then the relations among these quantities.  

$$h = \frac{gT^2}{8} \quad \dots\dots\dots (1); \quad gT^2 = 2R \tan \alpha \quad \dots\dots\dots (2);$$

$$R \tan \alpha = 4h \quad \dots\dots\dots (3)$$
- For a given initial velocity, to get the same horizontal range, there are two angles of projection  $\alpha$  and  $90^\circ - \alpha$
- The equation to the parabola traced by a body projected horizontally from the top of a tower of height  $y$ , with a velocity  $u$  is  $y = gx^2/2u^2$ , where  $x$  is the horizontal distance covered by it from the foot of the tower.

- At any instant if  $v$  is the velocity of projectile making angle  $\beta$  with the horizontal, then  

$$v_x = v \cos \beta = u \cos \theta \text{ and } v_y = v \sin \beta = u \sin \theta - gt$$
- Equation of trajectory is  $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$ , which is parabola.
- Maximum height is equal to  $n$  times the range when the projectile is launched at an angle  $\theta = \tan^{-1}(4n)$ .
- In a uniform circular motion, velocity and acceleration are constant only in magnitude. Their directions change.
- In a uniform circular motion, the kinetic energy of the body is a constant.  $W = 0, \vec{a} \neq 0, \vec{p} \neq \text{constant}, \vec{L} = \text{constant}$
- $a_r = \omega^2 r = \frac{v^2}{r} = \omega v$  (Always applicable)
- $a_r = 4\pi^2 n^2 r = \frac{4\pi^2}{T^2} r$  (Applicable in uniform circular motion)  
 $n = \text{frequency of rotation, } T = \text{time period of rotation.}$   

$$\vec{a}_r = \vec{\omega} \times \vec{v}$$

### LAWS OF MOTION

- Newton's second law :  $\vec{F} = m\vec{a}, \vec{F} = d\vec{p} / dt$
- Impulse :  $\Delta\vec{p} = \vec{F} \Delta t, \vec{p}_2 - \vec{p}_1 = \int_{t_1}^{t_2} \vec{F} dt$
- Newton's third law :  $\vec{F}_{12} = -\vec{F}_{21}$
- Frictional force  $f_s \leq (f_s)_{\max} = \mu_s R; f_k = \mu_k R$
- Circular motion with variable speed. For complete circle, the string must be taut in the highest position,  $u^2 \geq 5g\ell$  where  $\ell$  is the length of string.  
 Circular motion ceases at the instant when the string becomes slack, i.e. when  $T = 0$ , range of values of  $u$  for

which the string does go slack is  $\sqrt{2g\ell} < u < \sqrt{5g\ell}$ .

- Conical pendulum :  $\omega = \sqrt{g/h}$  where h is height of a point of suspension from the centre of circular motion.
- The acceleration of a lift

$a = \frac{\text{actual weight} - \text{apparent weight}}{\text{mass}}$ , where the

weights is in N. if 'a' is positive the lift is moving down, and if it is negative the lift is moving up.

### WORK, ENERGY AND POWER

- $e = -\frac{\text{velocity of separation of colliding bodies}}{\text{velocity of approach of colliding bodies}}$
- The total momentum of a system of particles is a constant in the absence of external forces.

### SYSTEM OF PARTICLES AND ROTATIONAL MOTION

- The centre of mass of a system of particles is defined as

the point whose position vector is  $R = \frac{\sum m_i r_i}{M}$  The centre

of gravity of an extended body is that point where the total gravitational torque on the body is zero. The centre of gravity of a body coincides with its centre of mass only if the gravitational field does not vary from one part of the body to the other.

- The angular momentum of a system of n particles about

the origin is  $L = \sum_{i=1}^n r_i \times p_i$

The torque or moment of force on a system of n particles

about the origin is  $\tau = \sum_i r_i \times F_i$

- The moment of inertia of a rigid body about an axis is defined by the formula  $I = \sum m_i r_i^2$  where  $r_i$  is the perpendicular distance of the  $i^{\text{th}}$  point of the body from the

axis. The kinetic energy of rotation is  $K = \frac{1}{2} I \omega^2$

- The theorem of parallel axes :  $I_z = I_c + Ma^2$ ,  
Theorem of perpendicular Axes :  $I_z = I_x + I_y$ .
- For rolling motion without slipping  $v_{cm} = R\omega$ , where  $v_{cm}$  is the velocity of translation (i.e., of the centre of mass), R is the radius and m is the mass of the body. The kinetic energy of such a rolling body is the sum of kinetic energies

of translation and rotation :  $K = \frac{1}{2} mv_{cm}^2 + \frac{1}{2} I \omega^2$

- A rigid body is in mechanical equilibrium if

(a) It is in translational equilibrium if the total external

force on it is zero :  $\sum F_i = 0$ .

(b) It is in rotational equilibrium i.e., the total external torque on it is zero :  $\sum_i \tau_i = \sum r_i \times F_i = 0$ .

### GRAVITATION

- The acceleration due to gravity.

(a) At a height  $h$  above the Earth's surface

$$g(h) = \frac{GM_E}{(R_E + h)^2} = \frac{GM_E}{R_E^2} \left( 1 - \frac{2h}{R_E} \right) \text{ for } h \ll R_E$$

(b) at depth  $d$  below the Earth's surface is

$$g(d) = \frac{GM_E}{R_E^2} \left( 1 - \frac{d}{R_E} \right)$$

- The gravitational potential energy

$$V = -\frac{Gm_1 m_2}{r} + \text{constat}$$

- The escape speed from the surface of the Earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E} \text{ and has value of } 11.2 \text{ km s}^{-1}.$$

- A geostationary (geosynchronous communication) satellite moves in a circular orbit in the equatorial plane at a approximate distance of  $4.22 \times 10^4$  km from the Earth's centre.

- Whenever force responsible for orbital motion obeys inverse square law, then only square of time period is directly proportional to cube of average distance between planet and sun.

$$T^2 \propto a^3; \frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}$$

Applicable only when both planets revolve around same mass. Length of semi major axis is the average distance between sun and planet during its complete orbital motion.

### MECHANICAL PROPERTIES OF FLUIDS

- Pascal's law** : A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel.
- Bernoulli's principle states that as we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume ( $\rho v^2 / 2$ ) and the potential energy per unit volume ( $\rho gy$ ) remains a constant.

$$P + \rho v^2 / 2 + \rho gy = \text{constant}$$

- Surface tension is a force per unit length (or surface energy per unit area) acting in the plane of interface between the

liquid and the bounding surface.

- Stokes' law state that the viscous drag force  $F$  on a sphere of radius  $a$  moving with velocity  $v$  through a fluid of viscosity  $\eta$  is,  $F = 6\pi\eta av$ .
- The surface tension of a liquid is zero at boiling point. The surface tension is zero at critical temperature.
- If a drop of water of radius  $R$  is broken into  $n$  identical drops, the work done in the process is  $4\pi R^2 S(n^{1/3} - 1)$
- Two capillary tubes each of radius  $r$  are joined in parallel. The rate of flow of liquid is  $Q$ . If they are replaced by single capillary tube of radius  $R$  for the same rate of flow then  $R = 2^{1/4} r$ .
- If radius of a drop is doubled its terminal velocity increases to four times.

### THERMAL PROPERTIES OF MATTER

- The coefficient of linear expansion ( $\alpha_l$ ), superficial expansion ( $\beta$ ) and volume expansion ( $\alpha_v$ ) are defined by the relations:

$$\frac{\Delta \ell}{\ell} = \alpha_l \Delta T; \frac{\Delta A}{A} = \beta \Delta T; \frac{\Delta V}{V} = \alpha_v \Delta T$$

where  $\Delta \ell$  and  $\Delta V$  denote the change in length  $\ell$  and volume  $V$  due to change of temperature  $\Delta T$ . The relation between them is :

$$\alpha_v = 3\alpha_l; \beta = 2\alpha_l$$

- In conduction, heat is transferred between neighbouring parts of a body through molecular collisions, without any flow of matter. For a bar of length  $L$  and uniform cross section  $A$  with its ends maintained at temperature  $T_C$  and  $T_D$ , then rate of flow of heat  $H$  is :

$$K = KA \frac{T_C - T_D}{L}, \text{ where } K \text{ is the thermal conductivity of the material of the bar.}$$

- Convection involves flow of matter within a fluid due to unequal temperatures of its parts.
- Stefan's law of radiation : The energy emitted by a black body per unit area per second is directly proportional to the fourth power of its Kelvin temperature.  $E = \sigma T^4$ , where the constant  $\sigma$  is known as Stefan's constant.
- Newton's Law of cooling :  $\frac{dQ}{dt} = -k(T_2 - T_1)$ ; where  $T_1$  is the temperature of the surrounding medium and  $T_2$  is the temperatures of the body.

### THERMODYNAMICS

- The first law of thermodynamics : It states that

$\Delta Q = \Delta U + \Delta W$ , where  $\Delta Q$  is the heat supplied to the system,  $\Delta W$  is the work done by the system and  $\Delta U$  is the change in internal energy of the system.

If  $Q > 0$ , heat is added to the system, if  $Q < 0$ , heat is removed to the system, if  $W > 0$ , Work is done by the system, if  $W < 0$ , Work is done on the system quantity.

- In an isothermal expansion of an ideal gas from volume  $V_1$  to  $V_2$  at temperature  $T$  the heat absorbed ( $Q$ ) equals the work done ( $W$ ) by the gas, each given by

$$Q = W = nRT \ln \left( \frac{V_2}{V_1} \right).$$

- In an adiabatic process of an ideal gas  $PV^\gamma = \text{constant}$ ,

$$\text{where } \gamma = \frac{C_p}{C_v}.$$

Work done by an ideal gas in an adiabatic change of state

$$\text{from } (P_1, V_1, T_1) \text{ to } (P_2, V_2, T_2) \text{ is } W = \frac{nR(T_1 - T_2)}{\gamma - 1}$$

- The efficiency of a Carnot engine is given by  $\eta = 1 - \frac{T_2}{T_1}$

### KINETIC THEORY

- Kinetic theory of an ideal gas gives the relation  $P = \frac{1}{3} n m \bar{v}^2$ , where  $n$  is number density of molecules,  $m$  the mass of the molecule and  $\bar{v}^2$  is the mean of squared speed. Combined with the ideal gas equation it yields a kinetic interpretation of temperature.

$$\frac{1}{2} n m \bar{v}^2 = \frac{3}{2} k_B T, v_{\text{rms}} = (\bar{v}^2)^{1/2} = \sqrt{\frac{3k_B T}{m}}$$

- The law of equipartition of energy is stated thus: the energy for each degree of freedom in thermal equilibrium is  $1/2 (k_B T)$

- The translational kinetic energy  $E = \frac{3}{2} k_B N T$ . This leads to a relation  $PV = \frac{2}{3} E$ .

- Speed of sound in a gas  $v_s = \sqrt{\frac{\gamma RT}{M}}$ ,  $\frac{v_s}{v_{\text{rms}}} = \sqrt{\frac{\gamma}{3}}$  i.e.,

$$v_s \approx v_{\text{rms}}$$

### OSCILLATIONS

- The particle velocity and acceleration during SHM as function of time are given by

$$v(t) = -\omega A \sin(\omega t + \phi) \text{ (velocity),}$$

$$a(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t) \text{ (acceleration)}$$

where  $x(t) = A \cos(\omega t + \phi)$

Velocity amplitude  $v_m = \omega A$ . and acceleration amplitude

$$a_m = \omega^2 A$$

2. A particle of mass  $m$  oscillating under the influence of a Hook's law restoring force given by  $F = -kx$  exhibits

simple harmonic motion with  $\omega = \sqrt{\frac{k}{m}}$  (angular frequency),

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ (period)}$$

Such a system is also called a linear oscillator.

3. A body of mass  $M$  is suspended from a spring whose force constant is  $K$  and mass is  $m$ . The time period of this system

$$\text{will be } 2\pi\sqrt{\frac{(M+m/3)}{k}}$$

4. Time period for conical pendulum  $T = 2\pi\sqrt{\left(\frac{\ell \cos\theta}{g}\right)}$

where  $\theta$  is angle between string & vertical.

### WAVES

1. The speed of a transverse wave on a stretched string is set by the properties of the string. The speed on a string with tension  $T$  and linear mass density  $\mu$   $v = \sqrt{T/\mu}$ .

2. Sound waves are longitudinal mechanical waves that can travel through solids, liquids, or gases. The speed  $v$  of sound wave in a fluid having bulk modulus  $B$  and density  $\rho$  is  $v = \sqrt{B/\rho}$ .

The speed of longitudinal waves in a metallic bar (stretched wire) is  $v = \sqrt{Y/\rho}$

For gases, since  $B = \gamma P$  (Adiabatic bulk modulus of elasticity), the speed of sound is  $v = \sqrt{\gamma P/\rho}$

3. Beats arise when two waves having slightly different frequencies,  $f_1$  and  $f_2$  and comparable amplitudes, are superposed. The beat frequency is  $f_{\text{beat}} = f_1 - f_2$
4. The Doppler effect is a change in the observed frequency of a wave when the source  $S$  and the observer  $O$  moves relative to the medium. For sound the observed frequency  $f$  is given in term of the source frequency  $f_0$  by

$$f = f_0 \left( \frac{v \pm v_o}{v \pm v_s} \right)$$

Here  $v$  is the speed of sound through the medium,  $v_o$  is the velocity of observer relative to the medium, and  $v_s$  is the source velocity relative to the medium. In using this formula, velocities in the direction  $OS$  should be treated

as positives and those opposite to it should be taken to be negative.

### ELECTROSTATICS

1. Coulomb's Law :  $\vec{F}_{21}$  = force on  $q_2$  due to  $q_1 = \frac{k(q_1q_2)}{r_{21}^2} \hat{r}_{21}$   
where  $\hat{r}_{21}$  is a unit vector in the direction from  $q_1$  to  $q_2$  and  $k = \frac{1}{4\pi\epsilon_0}$  is the constant of proportionality.

2. Electric field due to a point charge  $q$  has a magnitude  $|q|/4\pi\epsilon_0 r^2$  it is radially outwards from  $q$ , if  $q$  is positive, and radially inwards if  $q$  is negative. Like coulomb force, electric field also satisfies superposition principle.

3. An electric dipole is a pair of equal and opposite charges  $q$  and  $-q$  separated by some distance  $2a$ . Its dipole moment vector  $\vec{p}$  has magnitude  $2qa$  and is in the direction of the dipole axis from  $-q$  to  $q$ .

Field of an electric dipole in its equatorial plane (i.e., the plane perpendicular to its axis and passing through its centre) at a distance  $r$  from the centre.

$$\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 (a^2 + r^2)^{3/2}} = \frac{-\vec{p}}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

Dipole electric field on the axis at a distance  $r$  from the centre:

$$\vec{E} = \frac{2\vec{p}r}{4\pi\epsilon_0 (r^2 - a^2)^2} = \frac{2\vec{p}}{4\pi\epsilon_0 r^3} \quad \text{for } r \gg a$$

The  $1/r^3$  dependence of dipole electric fields should be noted in contrast to the  $1/r^2$  dependence of electric field due to a point charge. In a uniform electric field  $\vec{E}$ , a dipole experiences a torque  $\vec{\tau}$  given by  $\vec{\tau} = \vec{p} \times \vec{E}$  but experiences zero net force

4. The flux  $\Delta\phi = \vec{E} \Delta\vec{S}$
5. Gauss's law : The flux of electric field passing through any closed surface  $S$  is  $1/\epsilon_0$  times the total charge enclosed by  $S$ .

the law is especially useful in determining electric field  $\vec{E}$ , when the source distribution has simple symmetry.

- (a) Thin infinitely long straight wire of uniform linear charge

$$\lambda : \vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{n}$$

where  $r$  is the perpendicular distance of the point from the wire and  $\hat{n}$  is the radial unit vector in the plane normal to the wire passing through the point.

(b) Infinite thin plane sheet of uniform surface charge

$$\text{density } \sigma: \vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

(c) Thin spherical shell of uniform surface charge density

$$\sigma: \vec{E} = \frac{\sigma}{4\pi\epsilon_0 r^2} \hat{r} \quad (r \geq R); \vec{E} = 0 \quad (r < R)$$

6. Potential :  $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

For a charge configuration  $q_1, q_2, \dots, q_n$  with position vectors  $r_1, r_2, \dots, r_n$ , the potential at a point P is given by the

superposition principle  $V = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \dots + \frac{q_n}{r_{np}} \right)$

where  $r_{ip}$  is the distance between  $q_i$  and P, and so on.

7. The electrostatic potential at a point with position vector  $\vec{r}$  due to point dipole of dipole moment  $\vec{p}$  placed at the

origin is  $V(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{r^3}$

8. An equipotential surface is a surface over which potential has a constant value. For a point charge, concentric spheres centred at a location of the charge are equipotential surfaces. The electric field  $\vec{E}$  at a point is perpendicular to the equipotential surface through the point.  $\vec{E}$  is in the direction of the steepest decrease of potential.

9. Potential energy stored in a system of charges is the work done (by an external agency) in assembling the charges at their locations. Potential energy of two charges  $q_1, q_2$  at r

distance is given by  $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$ , where r is distance between  $q_1$  and  $q_2$ .

10. Capacitance is defined by  $C = Q/V$ , where Q is the charge on positive plate and V is the potential difference between plates. C is determined purely geometrically, by the shapes, sizes and relative positions of the two plates. The unit of capacitance is farad;  $1 \text{ F} = 1 \text{ C V}^{-1}$ . For a parallel plate capacitor (with vacuum between the plates),

$C = \epsilon_0 \frac{A}{d}$ , where A is the area of each plate and d the separation between them.

11. The energy U stored in a capacitor of capacitance C, with charge Q and voltage V is

$$U = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

12. For capacitors in the series combination, the total capacitance

$$C \text{ is given by } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

In the parallel combination, the total capacitance C is given by  $C = C_1 + C_2 + C_3 + \dots$

where  $C_1, C_2, C_3, \dots$  are individual capacitances.

### CURRENT ELECTRICITY

1. Current density  $j$  gives the amount of charge flowing per second per unit area normal to the flow,  $\vec{j} = nq\vec{v}_d$

2. Equation  $\vec{E} = \rho\vec{j}$  another statement of Ohm's law, i.e., a conducting material obeys Ohm's law when the resistivity of the material does not depend on the magnitude and direction of applied electric field.

3. (a) Total resistance R of n resistors connected in series is given by  $R = R_1 + R_2 + \dots + R_n$

(b) Total resistance R of n resistors connected in parallel

$$\text{is given by } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

where  $R_1, R_2, \dots, R_n$  are individual resistance.

4. Kirchhoff's Rules – (a) Junction Rule : At any junction of circuit element, the sum of currents entering the junction is equal to the sum of currents leaving it. (b) Loop Rule: the algebraic sum of the changes in potential in any closed loop is zero.

5. The Wheatstone bridge is an arrangement of four resistances –  $R_1, R_2, R_3, R_4$ . The null-point condition is

given by  $\frac{R_1}{R_2} = \frac{R_3}{R_4}$  using which the value of one resistance can be determined, knowing the other three resistance.

6. The potentiometer is a device to compare potential differences. Since the method involves a condition of no current flow, the device can be used to measure potential difference; internal resistance of a cell and compare emf's

of two sources.  $r = R \left( \frac{\ell_1}{\ell_2} - 1 \right)$

7. RC circuit: During charging:  $q = CE(1 - e^{-t/RC})$

During discharging:  $q = q_0 e^{-t/RC}$

### MAGNETISM

1. The total force on a charge q moving with velocity v in the presence of electric and magnetic field  $\vec{E}$  and  $\vec{B}$ , respectively is called the Lorentz force. It is given by the expression:  $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$ .

2. A straight conductor of length  $\ell$  and carrying a steady current I experiences a force  $\vec{F}$  in a uniform external

magnetic field  $\vec{B}$ ,  $\vec{F} = I \vec{\ell} \times \vec{B}$ , the direction of  $\ell$  is given by the direction of the current.

3. The Biot-Savart law asserts that the magnetic field  $d\vec{B}$  due to an element  $d\vec{\ell}$  carrying a steady current  $I$  at a point  $P$  at a distance  $r$  from the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \vec{r}}{r^3}.$$

To obtain the total field at  $P$ , we must integrate this vector expression over the entire length of the conductor.

4. Magnetic field due to straight current carrying conductor

$\vec{B} = \frac{\mu_0 I}{4\pi a} (\sin \theta_1 + \sin \theta_2)$ , where  $\theta_1$  and  $\theta_2$  are the angles between the line joining the point to the ends of conductor and perpendicular through the point to the conductor.

5. The magnitude of the magnetic field due to a circular coil of radius  $R$  carrying a current  $I$  at an axial distance  $x$  from the centre is

$$B = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}.$$

6. The magnitude of the field  $B$  inside a long solenoid carrying a current  $I$  is:  $B = \mu_0 nI$ , where  $n$  is the number of

turns per unit length. For a toroid,  $B = \frac{\mu_0 NI}{2\pi r}$ , where  $N$  is the total number of turns and  $r$  is the mean radius.

7. Ampere's Circuital Law: Let an open surface  $S$  be bounded by a loop  $C$ . Then the Ampere's law states that

$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I$ , where  $I$  refers to the current passing through  $S$ .

8. force per unit length between two long parallel wires carrying currents  $I_1$ ,  $I_2$  and separated by distance  $a$  in a

free space or air  $F = \frac{\mu_0 I_1 I_2}{2\pi a} N m^{-1}$ .

The force is attractive if currents are in the same direction and repulsive currents are in the opposite direction.

9. For current carrying coil  $\vec{M} = NI\vec{A}$ ; torque  $= \vec{\tau} = \vec{M} \times \vec{B}$

10. Consider a material placed in an external magnetic

field  $\vec{B}_0$ . The magnetic intensity is defined as,  $\vec{H} = \frac{\vec{B}_0}{\mu_0}$ .

The magnetisation  $\vec{M}$  of the material is its dipole moment per unit volume. The magnetic field  $B$  in the material is,  $\vec{B} = \mu_0(\vec{H} + \vec{M})$

11. For a linear material  $\vec{M} = \chi_m \vec{H}$ . So that  $\vec{B} = \mu \vec{H}$  and  $\chi_m$  is called the magnetic susceptibility of the material. The three quantities,  $\chi_m$ , the relative magnetic permeability  $\mu_r$ , and the magnetic permeability  $\mu$  are related as follows.

$$\mu = \mu_0 \mu_r; \mu_r = 1 + \chi_m.$$

### ELECTRO MAGNETIC INDUCTION

1. The magnetic flux through a surface of area  $A$  placed in a uniform magnetic field  $\vec{B}$  is defined as

$\phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta$ , where  $\theta$  is the angle between  $\vec{B}$  and  $\vec{A}$ .

2. Faraday's laws of induction imply that the emf induced in a coil of  $N$  turns is directly related to the rate of change of

flux through it  $\varepsilon = -N \frac{d\phi_B}{dt}$

3. When a metal rod of length  $\ell$  is placed normal to a uniform magnetic field  $B$  and moved with a velocity  $v$  perpendicular to the field, the induced emf (called motional emf) across its ends is  $\varepsilon = B\ell v$

4. When an current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by,

$\varepsilon = -L \frac{dI}{dt}$   $L$  is the self inductance of the coil, its a measure

of the inertia of the coil against the change of current through it.

5. A changing current in a coil (coil 2) can induce an emf in a nearby coil (coil 1). This relation is given by,

$\varepsilon_1 = -M_{12} \frac{dI_2}{dt}$ , The quantity  $M_{12}$  is called mutual

inductance of coil 1 with respect to coil 2.  $M_{12} = K\sqrt{L_1 L_2}$ .

6. LR circuit: For growth current,  $i = i_0[1 - e^{-Rt/L}]$

For decay of current,  $i = i_0 e^{-Rt/L}$

### ALTERNATING CURRENT

1. For an alternating current  $i = i_m \sin \omega t$  passing through a resistor  $R$ , the average power loss  $P$  (averaged over a cycle) due to joule heating is  $(1/2)i^2 m^R$ . To express it in the same form as the dc power ( $P = I^2 R$ ), a special value of current is used. It is called root mean square (rms) current and is

denoted by  $I = \frac{i_m}{\sqrt{2}} = 0.707 i_m$

The average power loss over a complete cycle is given by  $P = VI \cos \phi$ . the term  $\cos \phi$  is called the power factor.

2. An ac voltage  $v = v_m \sin \omega t$  applied to a pure inductor  $L$ , drive s a current in the inductor  $i = i_m \sin (\omega t - \pi/2)$ , where  $i_m = v_m/X_L$ .  $X_L = \omega L$  is called inductive reactance. The current in the inductor lags the voltage by  $\pi/2$ . The average power supplied to an inductor over one complete cycle is zero.

An ac voltage  $v = v_m \sin \omega t$  applied to a capacitor drives

a current in the capacitor:  $i = i_m \sin(\omega t + \pi/2)$ . Here,

$$i_m = \frac{v_m}{X_C}, X_C = \frac{1}{\omega C} \text{ is called capacitive reactance.}$$

3. An interesting characteristic of a series RLC circuit is the phenomenon of resonance. The circuit exhibits resonance, i.e., the amplitude of the current is maximum at the resonant

$$\text{frequency } \omega_0 = \frac{1}{\sqrt{LC}} (X_L = X_C).$$

The equality factor  $Q$  defined by  $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$  is an indicator of the sharpness of the resonance, the higher value of  $Q$  indicating sharper peak in the current.

### RAY OPTICS

1. For a prism of the angle  $A$ , of refractive index  $n_2$  placed in a medium of refractive index  $n_1$ ,

$$n_{21} = \frac{n_2}{n_1} = \frac{\sin[(A + D_m)/2]}{\sin(A/2)}, \text{ where } D_m \text{ is the angle of}$$

minimum deviation. Dispersion is the splitting of light into its constituent colours. The deviation is maximum for violet and minimum for red. Dispersive power  $\omega$  is the ratio of angular dispersion ( $\delta_v - \delta_r$ ) to the mean deviation  $\delta$ .

$\omega = \frac{\delta_v - \delta_r}{\delta}$ , where  $\delta_v, \delta_r$  are deviation of violet and red respectively and  $\delta$  the deviation of mean ray (usually yellow).

2. for refraction through a spherical interface (from medium 1 to 2 of refractive index  $n_1$  and  $n_2$  respectively)

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}. \text{ Thin lens formula } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \text{ Lens}$$

$$\text{maker's formula: } \frac{1}{f} = \frac{(n_2 - n_1)}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

The power of a lens  $P = 1/f$ . The SI unit for power of a lens is dioptre (D):  $1D = 1 \text{ m}^{-1}$ .

If several thin lenses of focal length  $f_1, f_2, f_3, \dots$  are in contact, the effective focal length of their combination, is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \dots$$

The total power of a combination of several lenses is

$$P = P_1 + P_2 + P_3 + \dots$$

If distance between lens is  $d$  then power of combination

$$= P_1 + P_2 - d \times P_1 P_2$$

Chromatic aberration is the colouring of image produced by lenses. This can be avoided by combining a convex and a concave lens of focal length  $f_1$  and  $f_2$  and dispersive

powers  $\omega_1, \omega_2$  respectively satisfying the equation

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \text{ or in terms of powers } \omega_1 P_1 + \omega_2 P_2 = 0.$$

### WAVE OPTICS

1. Young's double slit of separation  $d$  gives equally spaced fringes of angular separation  $\lambda/d$ . The source, mid-point of the slits, and central bright fringe lie in a straight line. An extended source will destroy the fringes if it subtends angle more than  $\lambda/d$  at the slits. The resultant intensity of two waves of intensity  $I_0/4$  of phase difference  $\phi$  at any points is given by

$$I = I_0 \cos^2 \left[ \frac{\phi}{2} \right], \text{ where } I_0 \text{ is the maximum intensity.}$$

$$\text{Condition for dark band : } \delta = (2n-1) \frac{\lambda}{2},$$

$$\text{For bright band : } \delta = n\lambda, \text{ Fringe width } \beta = \frac{D\lambda}{d}$$

### MODERN PHYSICS

1. Einstein's photoelectric equation is
- $$\frac{1}{2} m v_{\max}^2 = v_0 e = h\nu - \phi_0 = h(\nu - \nu_0)$$
2. The nuclear mass  $M$  is always less than the total mass,  $\Sigma m_i$ , of its constituents. The difference in mass of a nucleus and its constituents is called the mass defect,
- $$\Delta M = (Zm_p + (A - Z)m_n) - M; \Delta E_b = \Delta M c^2.$$
- 1 amu = 931 MeV
3.  $E_n = -\frac{Z^2}{n^2} \times 13.6 \text{ eV}$  (for hydrogen like atom)
4. Bragg's law :  $2d \sin \theta = n\lambda$
5. Law of radioactive decay :  $N = N_0 e^{-\lambda t}$ .
- $$\text{Activity} = \frac{dN}{dt} = -\lambda N \text{ (unit is Becquerel)}$$
6. Half time period,  $T_{1/2} = \frac{0.693}{\lambda}$
7. X-ray :  $\lambda_{\min} = \frac{12400}{V} \text{ \AA}$
- Characteristics X-ray:  $\lambda_{K_\alpha} < \lambda_{L_\alpha}$
- Moseley law :  $\nu = a(Z - b)^2$

Engineering  
IIT-JEE

Where is  
my career?

Medical  
PMT/NEET

Where are you going? Where do you want to be?  
inspiring your future



**APEX EDUCATION**

**IIT-JEE / PMT / FOUNDATION / CBSE**

Success through hard work & disciplined study ...

## PHYSICAL CHEMISTRY

### ATOMIC STRUCTURE

- $r_n = \frac{n^2 h^2}{4\pi^2 m Z e^2} = 0.529 \left( \frac{n^2}{Z} \right) \text{Å}, r_n = n^2 \times r_1$
- $E_T = -KE = \frac{PE}{2} = -13.6 \frac{Z^2}{n^2} \text{eV}$
- $\Delta E = \frac{hc}{\lambda} = \frac{2\pi^2 m e^4}{h^2} \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$
- $\bar{\nu} = \frac{1}{\lambda} = RZ^2 \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right] [R = 1.0968 \times 10^7 \text{m}^{-1}]$
- Total no. of spectrum lines =  $\frac{n(n-1)}{2}$
- Heisenberg Uncertainty Principle  $(\Delta x)(\Delta p) \geq h / 4\pi$
- Moseley's law  $\sqrt{\nu} = a(Z-b), E_n = -\frac{13.7}{n^2} \text{eV / atom}$
- Nodes  $(n-1) = \text{total nodes}, \ell = \text{angular nodes},$   
 $(n - \ell - 1) = \text{Radial nodes}$
- Photoelectric effect :  $h\nu = h\nu_0 + \frac{1}{2}mv^2$
- Orbital angular momentum:  $\sqrt{\ell(\ell+1)} \frac{h}{2\pi}$

### CHEMICAL BONDING

- % ionic character =  $\frac{\text{Actual dipole moment}}{\text{Calculated dipole moment}} \times 100$
- Fajan's Factors: Following factors are helpful in inducing covalent character in Ionic compounds
  - Small cation
  - Big anion
  - High charge on cation
  - High charge on anion
  - Cation having pseudo inert gas configuration ( $ns^2p^6d^{10}$ )

e.g.  $\text{Cu}^+, \text{Ag}^+, \text{Zn}^{2+}, \text{Cd}^{2+}$

- $-\Delta H_f = H_s + \frac{1}{2}H_d + IE + \Delta H_{EG} - E_L$
- M.O. theory:
  - Bond order =  $\frac{1}{2}(N_b - N_a)$
  - Higher the bond order, higher is the bond dissociation energy, greater is the stability, shorter is the bond length.
  - Species      Bond order      Magnetic properties
 

$\text{H}_2$	1	Diamagnetic
$\text{H}_2^+$	0.5	Paramagnetic
$\text{Li}_2$	1	Diamagnetic
- $Q = \frac{1}{2}[V + SA - (\pm q)]$
- Formal charge =  $V - \left( L + \frac{1}{2}S \right)$
- VSEPR theory
  - (LP-LP) repulsion > (LP-BP) > (BP-BP)
  - For  $\text{NH}_3 \rightarrow$  Bond Angle  $106^\circ 45'$  in case of water it decreases to  $104^\circ 27'$  because  $\text{H}_2\text{O}$  molecule contains 2LP and 2BP whereas  $\text{NH}_3$  has 1LP and 3BP.
- Bond angle:  
Decrease in bond angle down the gp is due to LP-BP repulsion  
(a)  $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3$ , (b)  $\text{H}_2\text{O} > \text{H}_2\text{S} > \text{H}_2\text{Se}$

### CHEMICAL EQUILIBRIUM

- $K_p = K_c (RT)^{\Delta n_g}$  where  $\Delta n_g = n_p - n_r$
- Free Energy change ( $\Delta G$ )
  - If  $\Delta G = 0$  then reversible reaction would be in equilibrium.
  - If  $\Delta G = (+)$  ve the equilibrium will be displaced in



backward direction.

(c) If  $\Delta G = (-)$  ve then equilibrium will displace in forward direction.

- $K_c$  unit  $\rightarrow$  (moles/lit) <sup>$\Delta n$</sup> ,
  - $K_p$  unit  $\rightarrow$  (atm) <sup>$\Delta n$</sup>
  - total moles at equilibrium = [total initial moles +  $\Delta n$ ]
  - time required to establish equilibrium  $\propto 1/K_c$
  - If in any heterogenous equilibrium solid substance is also present then its active mass & partial pressure is assumed 1.
- Le chatelier's principle
  - Increase of reactant conc. (Shift forward)
  - Decrease of reactant conc. (shift backward)
  - Increase of pressure (from more moles to less moles)
  - Decrease of pressure (from less moles to more moles)
  - For exothermic reaction decrease in temp. (Shift forward)
  - for endothermic increase in temp. ("Shift forward)

#### ACID BASE

- Lewis Acid ( $e^-$  pair acceptor)  $\rightarrow$   $CO_2$ ,  $BF_3$ ,  $AlCl_3$ ,  $ZnCl_2$ ,  $FeCl_3$ ,  $PCl_3$ ,  $PCl_5$ ,  $SiCl_4$ ,  $SF_6$ , normal cation
  - Lewis Base ( $e^-$  pair donor)  $NH_3$ ,  $ROH$ ,  $ROR$ ,  $H_2O$ ,  $RNH_2$ ,  $R_2NH$ ,  $R_3N$ , normal anion
- Dissociation of Weak Acid & Weak Base  $\rightarrow$ 
  - Weak Acid  $\rightarrow K_a = Cx^2/(1-x)$  or  $K_a = Cx^2$
  - Weak Base  $\rightarrow K_b = Cx^2/(1-x)$  or  $K_b = Cx^2$
- Buffer solution:**
  - Acidic  $\rightarrow pH = pK_a + \log \{Salt/Acid\}$  for Maximum Buffer action  $pH = pK_a$   
Range of Buffer  $pH = pK_a \pm 1$
  - Alkaline  $\rightarrow pOH = pK_b + \log \{Salt/Base\}$  for max.  
Buffer range for basic buffer =  $pK_b \pm 1$
  - Buffer Capacity =  $\frac{\text{Moles / lit of Acid or Base Mixed}}{\text{change in pH}}$

$$B = \frac{dCBOH}{dpH} = -\frac{dCHB}{dpH}$$

- Necessary condition for showing neutral colour of Indicator  $pH = pK_{In}$  or  $[HIn] = [In^-]$  or  $[InOH] = [In^+]$

#### IONIC EQUILIBRIUM

- Relation between ionisation constant ( $K_i$ ) & degree of ionisation ( $\alpha$ ):-

$$K_i = \frac{\alpha^2}{(1-\alpha)V} = \frac{\alpha^2 C}{(1-\alpha)} \quad (\text{Ostwald's dilution law})$$

It is applicable to weak electrolytes for which  $\alpha \ll 1$  then

$$\alpha = \sqrt{K_i V} = \sqrt{\frac{K_i}{C}} \quad \text{or } V \uparrow C \downarrow \alpha \uparrow$$

- Common ion effect : By addition of X mole/L of a common ion, to a weak acid (or weak base)  $\alpha$  becomes equal to  $\frac{K_a}{X}$  (or  $\frac{K_b}{X}$ ) [where  $\alpha$  = degree of dissociation]
- If solubility product = ionic product then the solution saturates.
  - If solubility product > ionic product then the solution is unsaturated and more of the substance can be dissolved in it.
  - If ionic product > solubility product the solution is super saturated (principle of precipitation).
- Salt of weak acid and strong base:  
 $pH = 0.5 (pK_w + pK_a + \log c)$   
Salt of weak base and strong acid:  
 $pH = 0.5 (pK_w - pK_b - \log c)$   
Salt of weak acid and weak base :  
 $pH = 0.5 (pK_w + pK_a - pK_b)$

#### CHEMICAL KINETICS

- Unit of Rate constant:  
 $K = \text{mol}^{1-\Delta n} \text{lit}^{\Delta n-1} \text{sec}^{-1}$
- First Order reaction:  
 $K = \frac{2.303}{t} \log_{10} \frac{a}{(a-x)}$  &  $t_{1/2} = \frac{0.693}{K}$   
 $[A]_t = [A]_0 e^{-kt}$
- Second Order Reaction:  
When concentration of A and B taking same.  
 $K_2 = \frac{1}{t} \left( \frac{x}{a(a-x)} \right)$   
when concentration of A and B are taking different.  
 $K_2 = \frac{2.303}{t(a-b)} \log \frac{b(a-x)}{a(b-x)}$
- Zero Order Reaction:  
 $K = \frac{a_0 - a_t}{t}$

$$x = Kt \text{ \& } t_{1/2} = \frac{a_0}{2K}$$

The rate of reaction is independent of the concentration of the reacting substance.

5. Arrhenius equation:

$$k = Ae^{-E_a/RT} \text{ \& } \text{slope} = \frac{-E_a}{2.303R}$$

when  $T \rightarrow \infty$ , then  $K = A (\because e^{-E_a/RT} = 1)$

$$6. \log \left( \frac{k_2}{k_1} \right) = \frac{E_a}{2.303R} \left( \frac{T_2 - T_1}{T_1 T_2} \right)$$

### OXIDATION-REDUCTION

- Oxidant itself is reduced (gives  $O_2$ )  
Or Oxidant  $\longrightarrow e^-$  (s) Acceptor  
Reductant itself is oxidised (given  $H_2$ )  
Or reductant  $\longrightarrow e^-$  (s) Donor
- (i) Strength of acid  $\propto$  O.N  
(ii) Strength of base  $\propto$  1 / O.N
- (a) Electro Chemical Series:- Li, K, Ba, Sr, Ca, Na, Mg, Al, Mn, Zn, Cr, Fe, Cd, Co, Ni, Sn, Pb,  $H_2$ , Cu,  $I_2$ , Hg, Ag,  $Br_2$ ,  $Cl_2$ , Pt, Au,  $F_2$ .  
(b) As we move from top to bottom in this series
  - Standard Reduction Potential  $\uparrow$
  - Standard Oxidation Potential  $\downarrow$
  - Reducing Capacity  $\downarrow$
  - $I_p \uparrow$
  - Reactivity  $\downarrow$
- (a) Formal charge = Group No. - [No of bonds + No. of non-bonded  $e^-$ s]  
(b) At anode  $\rightarrow$  Oxidation, Cathode  $\rightarrow$  Reduction

### VOLUMETRIC ANALYSIS

- Equivalent weight of element =  $\frac{\text{Atomic wt of the element}}{n \text{ factor}}$
- Equivalent weight of compound =  $\frac{\text{Formula wt of the compound}}{n \text{ factor}}$

- Equivalent wt. of an ion =  $\frac{\text{formula wt (or At. Wt.) of ion}}{\text{its valency}}$
- The law Dulong and Petit  
atomic wt.  $\times$  specific heat = 6.4
- Normality (N) =  $\frac{\text{number of equivalent of solute}}{\text{volume of solution in litres}}$
- Molarity (M) =  $\frac{\text{number of moles of solute}}{\text{volume of solution in litres}}$
- When a solution is diluted  
 $N_1 \times V_1 = N_2 \times V_2$   
(before dilution) (after dilution)
- Common acid-base indicators

Indicator	Colour in acidic medium	Colour in alkaline medium	pH range
Methyl orange	Pink	Yellow	3.0 - 4.4
Methyl red	Red	Yellow	4.2 - 6.2
Litmus	Red	Blue	5.5 - 7.5
Phenolphthalein	Colourless	Pink	8.3 - 9.8

### MOLE CONCEPT

- Mole concept  
GAM  $\equiv$  1 gm atom  $\equiv$   $6.02 \times 10^{23}$  atom,  
GMM = 1 gm molecule  $\equiv$   $6.02 \times 10^{23}$  molecules  
 $N_A = 6.02 \times 10^{23}$
- Moles (gases) at NTP =  $\frac{\text{volume(L)}}{22.4}$
- Molecular mass = 2  $\times$  vapour density

### CHEMICAL ENERGETICS

- First Law :  $\Delta E = Q + W$   
Expression for pressure volume work  $W = - P \Delta V$   
Maximum work in reversible expansion:  
 $W = -2.30nRT \log \frac{V_2}{V_1} = -2.30nRT \log \frac{P_1}{P_2}$
- Enthalpy and heat content :  $\Delta H = \Delta E + P \Delta V$   
 $[q_{(p)} = q_{(v)} + \Delta n_g RT] \quad \Delta H = \Delta E + \Delta n_g RT$   
 $[\Delta n_g = n_{p(g)} - n_{r(g)}]$
- Kirchoff's equation:  
 $\Delta E_{T_2} = \Delta E_{T_1} + \Delta C_v (T_2 - T_1) [\text{constan t V}]$

$$\Delta H_{T_2} = \Delta H_{T_1} + \Delta C_p(T_2 - T_1) \text{ [constant P]}$$

4. Entropy (s) : Measure of disorder or randomness

$$\Delta S = \sum S_p - \sum S_r$$

$$\Delta S = \frac{q_{rev}}{T} = 2.303 nR \log \frac{V_2}{V_1} = 2.030 nR \log \frac{P_1}{P_2}$$

5. Free energy change;  $\Delta G = \Delta H - T\Delta S$

$$\Delta G < 0 \text{ (spontaneous) [-ve]} \quad \Delta G = 0 \text{ (equilibrium)}$$

$$\Delta G > 0 \text{ (non-spontaneous) [+ ve]}$$

$$-\Delta G = W \text{ (maximum)} - P\Delta V$$

### ELECTRO CHEMISTRY

1.  $m = Z.I.t$

2. Degree of dissociation:  $\alpha = \frac{\lambda_{eq}}{\lambda_{\infty}^{eq}}$

3. Kohlrausch's law :  $\Lambda_m^0 = x\lambda_A^0 + y\lambda_B^0$

4. Nernst Equation

$$E = E^0 - \frac{0.0591}{n} \log_{10} \frac{[\text{Products}]}{[\text{Reactants}]}$$

$$\& E_{\text{Cell}}^0 = E_{\text{anode}}^0 + E_{\text{cathode}}^0 \quad \& K_{\text{eq.}} = \text{anti log} \left[ \frac{nFE^0}{0.0591} \right]$$

$$\Delta G = -nFE_{\text{cell}} \quad \& \Delta G^0 = -nFE_{\text{cell}}^0$$

$$-\Delta G^0 = 2.303RT \log K_c$$

$$\& W_{\text{max}} = +nFE^0 \quad \& \Delta G = \Delta H + T \left( \frac{\partial \Delta G}{\partial T} \right)$$

5. Calculation of pH of an electrolyte by using a calomel

electrode:  $\text{PH} = \frac{E_{\text{cell}} - 0.2415}{0.0591}$

### SOLUTION AND COLLIGATIVE PROPERTIES

1. Raoult's law

$$P = p_A + p_B = p_A^0 X_A + p_B^0 X_B$$

$$= (1 - X_B) p_A^0 + p_B^0 X_B = (p_B^0 - p_A^0) X_B + p_A^0$$

$$\frac{P_0 - P_s}{P_0} = \frac{n}{n+N} \quad \& \quad \frac{P_0 - P_s}{P_0} = \frac{w.m}{W.M}$$

2. Colligative properties
- $\propto$  Number of particles
  - $\propto$  Number of molecules (in case of nonelectrolytes)
  - $\propto$  Number of ions (in case of

electrolytes)

$\propto$  Number of moles of solute

$\propto$  Mole fraction of solute

3. Depression of freezing point,  $\Delta T_f = K_f m$

4. Elevation in boiling point with relative lowering of vapour pressure

$$\Delta T_b = \frac{1000k_b}{M_1} \left( \frac{p^0 - p}{p^0} \right) (M_1 = \text{mol. wt. of solvent})$$

5. Osmotic pressure (P) with depression in freezing point

$$\Delta T_f \cdot P = \Delta T_f \times \frac{dRT}{1000K_f}$$

6.  $i = \frac{\text{Normal molar mass}}{\text{Observed molar mass}} = \frac{\text{Observed colligative property}}{\text{Normal colligative property}}$

$$i = \frac{\text{Observed osmotic pressure}}{\text{Normal osmotic pressure}}$$

$$= \frac{\text{Actual number of particles}}{\text{No. of particles for no. ionisation}}$$

$$\text{degree of association (a)} = (1 - i) \frac{n}{n-1}$$

$$\& \text{degree of dissociation } (\alpha) = \frac{i-1}{n-1}$$

### NUCLEAR CHEMISTRY

1. Radius of the nucleus :  $R = R_0 A^{1/3}$

2. The amount N of the radioactive substance left after 'n'

$$\text{half lives} = \frac{N_0(\text{initial amount})}{2^n}$$

3. Half-life period  $t_{1/2} = \frac{0.693}{\lambda}$

4. Rate of disintegration:

$$-\frac{dN}{dt} = \lambda.N \quad \& \quad \lambda = \frac{2.303}{t} \log_{10} \frac{N_0}{N} \quad \text{or} \quad N = N_0 e^{-\lambda t}$$

5. Average life ( $t_{AV}$ ) =  $\frac{\text{Total life time of all the atoms}}{\text{Total number of atoms}}$

$$= \frac{\int_0^{\infty} t dN}{N_0} = \frac{1}{\lambda} = 1.44 t_{1/2}$$

### GASEOUS STATE

1. Ideal gas equation:  $PV = nRT$

(i)  $R = 0.0821 \text{ litre atm. K}^{-1} \text{ mol}^{-1}$

(ii)  $R = 62.4 \text{ litres mm Hg K}^{-1} \cdot \text{mol}^{-1}$

(iii)  $R = 8.314 \times 10^7 \text{ ergs K}^{-1} \cdot \text{mol}^{-1}$

(iv)  $R = 2 \text{ cal K}^{-1} \cdot \text{mol}^{-1}$

(v)  $R = 8.314 \text{ JK}^{-1} \cdot \text{mol}^{-1}$

## 2. Velocities related to gaseous state

$$\text{RMS velocity } C = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{d}}$$

$$\text{Average speed} = \sqrt{\frac{8RT}{\pi M}} \quad \& \quad \text{Most probable speed} =$$

$$\sqrt{\frac{2RT}{M}}$$

Average speed =  $0.9213 \times \text{RMS speed}$

RMS speed =  $1.085 \times \text{Average speed}$

MPS =  $.816 \times \text{RMS}$ ; RMS =  $1.224 \text{ MPS}$

MPS; A.V. speed : RMS =  $1 : 1.128 : 1.224$

3. Rate of diffusion  $\propto \frac{1}{\sqrt{\text{density of gas}}}$

## 4. Van der Waal's equation

$$\left( P + \frac{n^2 a}{V^2} \right) (V - nb) = nRT \quad \text{for } n \text{ moles}$$

5.  $Z$  (compressibility factor) =  $\frac{PV}{nRT}$ ;  $Z = 1$  for ideal gas

**SOLID AND LIQUID STATE**

## 1. Available space filled up by hard spheres (packing fraction)

Simple cubic =  $\frac{\pi}{6} = 0.52$

bcc =  $\frac{\pi\sqrt{3}}{8} = 0.68$       fcc =  $\frac{\pi\sqrt{2}}{6} = 0.74$

hcp =  $\frac{\pi\sqrt{2}}{6} = 0.74$       diamond =  $\frac{\pi\sqrt{3}}{6} = 0.34$

## 2. Radius ratio and co-ordination number (CN)

Limiting radius ratio	CN	Geometry
[0.155 – 0.225]	3	[plane triangle]
[0.255 – 0.414]	4	[tetrahedral]
[0.414 – 0.732]	6	[octahedral]
[0.732 – 1]	8	[bcc]

3. Atomic radius  $r$  and the edge of the unit cell.

Pure elements:

$$\text{Simple cubic} = r = \frac{a}{2} \quad \text{bcc } r = \frac{\sqrt{3}a}{4} \quad \text{fcc} = \frac{\sqrt{2}a}{4}$$

4. Relationship between radius of void ( $r$ ) and the radius of the sphere ( $R$ ):  $r$  (tetrahedral) =  $0.225 R$ ;  $r$  (octahedral) =  $0.414 R$ 

5. Paramagnetic: Presence of unpaired electrons [attracted by magnetic field]

6. Paramagnetic: Permanent magnetism [ $\uparrow\uparrow\uparrow\uparrow$ ]7. Antiferromagnetic: net magnetic moment is zero [ $\uparrow\downarrow\uparrow\downarrow$ ]8. Ferrimagnetic: net magnetic moment is three [ $\uparrow\downarrow\downarrow\uparrow\uparrow$ ]**INORGANIC CHEMISTRY****PERIODIC TABLE**

## 1. General electronic configuration (of outer orbits)

s-block  $ns^{1-2}$       p-block  $ns^2 np^{1-6}$ d-block  $(n-1) d^{1-10} ns^{1-2}$ f-block  $(n-2) s^2 p^6 d^{10} f^{1-14} (n-1) s^2 p^6 d^0$  or  $1 ns^2$ 

## 2. Properties      Pr(L TO R)      Gr(T to B)

(a) atomic radius      ↓      ↑

(b) ionisation potential      ↑      ↓

(c) electron affinity      ↑      ↓

(d) electro negativity      ↑      ↓

(e) metallic character or      ↓      ↑

electropositive character

3.  $EA \propto \frac{1}{\text{size}} \propto \text{nuclear charge}$ .

Second electron affinity is always positive.

Electron affinity of chlorine is greater than fluorine.

4. The first element of a group has similar properties with the second element of the next group. This is called diagonal relationship.

**EXTRACTIVE METALLURGY**

1. Floatation is a physical method of separating a mineral from the gangue depending on differences in their wettabilities by a liquid solution.

2. Roasting is the process of heating a mineral in the presence of air.

3. Calcination is the process of heating the ore in the absence of air.

4. Electrolytic reduction: Highly electropositive metals are extracted by the electrolysis of their oxides and hydroxides.

**s-BLOCK ELEMENTS**1. Atomic radii :  $\text{Li} < \text{Na} < \text{K} < \text{Rb} < \text{Cs}$ 2. Ionic radii :  $\text{Li}^+ < \text{Na}^+ < \text{K}^+ < \text{Rb}^+ < \text{Cs}^+$ 3. Electronegativity:  $\text{Li} > \text{Na} > \text{K} > \text{Rb} > \text{Cs}$

4. First ionization potential :  $\text{Li} > \text{Na} > \text{K} > \text{Rb} > \text{Cs}$
5. Melting point  $\text{Li} > \text{Na} > \text{K} > \text{Rb} > \text{Cs}$
6. Density :  $\text{Li} > \text{Na} < \text{K} > \text{Rb} > \text{Cs}$
7. Colour of the flame Li -red, Na-Golden, K-Violet, Rb-Red, Cs - Blue, Ca - Brick red, Sr-Blood red, Ba- Apple green
8. Rb and Cs show photoelectric effect.
9. Stability of hydrides :  $\text{LiH} > \text{NaH} > \text{KH} > \text{RbH} > \text{CsH}$
10. Basic nature of hydroxides:  $[\text{LiOH} < \text{NaOH} < \text{KOH} < \text{RbOH} < \text{CsOH}]$

**BORON FAMILY**

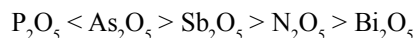
1. Stability of +3 oxidation state :  $\text{B} > \text{Al} > \text{Ga} > \text{In} > \text{Tl}$
2. Stability of +1 oxidation state :  $\text{Ga} < \text{In} < \text{Tl}$
3. Reducing nature :  $\text{Al} > \text{Ga} > \text{In} > \text{Tl}$
4. Basic nature of the oxides and hydroxides :  
 $\text{B} < \text{Al} < \text{Ga} < \text{In} < \text{Tl}$
5. Relative strength of Lewis acid:  $\text{BF}_3 < \text{BCl}_3 < \text{BBr}_3 < \text{BI}_3$

**CARBON FAMILY**

1. Reactivity :  $\text{C} < \text{Si} < \text{Ge} < \text{Sn} < \text{Pb}$
2. Metallic character :  $\text{C} < \text{Si} < \text{Ge} < \text{Sn} < \text{Pb}$
3. Acidic character of the oxides:  
 $\text{CO}_2 > \text{SiO}_2 > \text{GeO}_2 > \text{SnO}_2 > \text{PbO}_2$   
Weaker acidic (amphoteric)
4. Thermal stability and volatility of hydrides:  
 $\text{CH}_4 > \text{SiH}_4 > \text{GeH}_4 > \text{SnH}_4 > \text{PbH}_4$
5. Reducing nature of hydrides  
 $< \text{CH}_4 < \text{SiH}_4 < \text{GeCl}_4 < \text{SnH}_4 < \text{PbH}_4$
6. Thermal stability of tetrahalides  
 $\text{CCl}_4 > \text{SiCl}_4 > \text{GeCl}_4 > \text{SnCl}_4 > \text{PbCl}_4$
7. Thermal stability and volatility of tetrahalide with a common central atom  
 $\text{MF}_4 > \text{MCl}_4 > \text{MBr}_4 > \text{MI}_4$
8. Oxidising character of  $\text{M}^{+4}$  species  
 $\text{GeCl}_4 < \text{SnCl}_4 < \text{PbCl}_4$
9. Ease of hydrolysis of tetrahalides  
 $\text{SiCl}_4 > \text{GeCl}_4 > \text{SnCl}_4 > \text{PbCl}_4$
10. Reducing character of dihalides  
 $\text{GeCl}_2 > \text{SnCl}_2 > \text{PbCl}_2$

**NITROGEN FAMILY**

1. Acidic strength of trioxides :  $\text{N}_2\text{O}_3 > \text{P}_2\text{O}_3 > \text{As}_2\text{O}_3$
2. Acidic strength of pentoxides  
 $\text{N}_2\text{O}_5 > \text{P}_2\text{O}_5 > \text{As}_2\text{O}_5 > \text{Sb}_2\text{O}_5 > \text{Bi}_2\text{O}_5$
3. Acidic strength of oxides of nitrogen  
 $\text{N}_2\text{O} < \text{NO} < \text{N}_2\text{O}_3 < \text{N}_2\text{O}_4 < \text{N}_2\text{O}_5$
4. The stability of pentoxides



5. Basic nature, bond angle, thermal stability and dipole moment of hydrides  
 $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3 > \text{BiH}_3$
6. Reducing power, covalent nature of hydrides :  $\text{NH}_3 < \text{PH}_3 < \text{AsH}_3 < \text{SbH}_3 < \text{BiH}_3$
7. Stability of trihalides of nitrogen :  $\text{NF}_3 > \text{NCl}_3 > \text{NBr}_3$
8. Ease of hydrolysis of trichlorides  
 $\text{NCl}_3 > \text{PCl}_3 > \text{AsCl}_3 > \text{SbCl}_3 > \text{BiCl}_3$
9. Lewis acid strength of trihalides of P, As and Sb  
 $\text{PCl}_3 > \text{AsCl}_3 > \text{SbCl}_3$
10. Lewis acid strength among phosphorus trihalides  
 $\text{PF}_3 > \text{PCl}_3 > \text{PBr}_3 > \text{PI}_3$
11. Bond angle, among the halides of phosphorus  
 $\text{PF}_3 < \text{PCl}_3 < \text{PBr}_3 < \text{PI}_3$

**OXYGEN FAMILY**

1. Melting and boiling point of hydrides  
 $\text{H}_2\text{O} > \text{H}_2\text{Te} > \text{H}_2\text{Se} > \text{H}_2\text{S}$
2. Volatility of hydrides  
 $\text{H}_2\text{O} < \text{H}_2\text{Te} < \text{H}_2\text{Se} < \text{H}_2\text{S}$
3. Thermal stability of hydrides  
 $\text{H}_2\text{O} > \text{H}_2\text{S} > \text{H}_2\text{Se} > \text{H}_2\text{Te}$
4. Reducing nature of hydrides  
 $\text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$
5. Covalent character of hydrides  
 $\text{H}_2\text{O} < \text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$
6. Bond angle & Dipole moment of hydrides  
 $\text{H}_2\text{O} > \text{H}_2\text{S} > \text{H}_2\text{Se} > \text{H}_2\text{Te}$   
(104°) (92°) (91°) (90°)
7. Ease of hydrolysis of hexahalides:  $\text{SF}_6 > \text{SeF}_6 > \text{TeF}_6$
8. The acidic character of oxides (elements in the same oxidation state)  
 $\text{SO}_2 > \text{SeO}_2 > \text{TeO}_2 > \text{PoO}_2$   
 $\text{SO}_3 > \text{SeO}_3 > \text{TeO}_3$
9. Acidic character of oxide of a particular element (e.g.S)  
 $\text{SO}_2 < \text{SO}_3$
10. Stability of dioxides  
 $\text{SO}_2 > \text{TeO}_2 > \text{SeO}_2 > \text{PoO}_2$

**HALOGEN FAMILY**

1. Bond energy of halogens :  $\text{Cl}_2 > \text{Br}_2 > \text{F}_2 > \text{I}_2$
2. Bond length in  $\text{X}_2$  molecule :  $\text{F}_2 < \text{Cl}_2 < \text{Br}_2 < \text{I}_2$
3. Solubility of halogen in water :  $\text{F}_2 > \text{Cl}_2 > \text{Br}_2 > \text{I}_2$

- Oxidising power :  $F_2 > Cl_2 > Br_2 > I_2$
- Enthalpy of hydration of  $X^-$  ion:  $F^- > Cl^- > Br^- > I^-$
- Reactivity of halogens :  $F > Cl > Br > I$
- Ionic character of  $M-X$  bond in halides  
 $M-F > M-Cl > M-Br > M-I$
- Reducing character of  $X^-$  ion :  $I^- > Br^- > Cl^- > F^-$
- Thermal stability of hydrides :  $HF > HCl > HBr > HI$
- Acidic strength of halogen acids:  $HI > HBr > HCl > HF$
- Conjugate base strength of halogen acids  
 $I^- < Br^- < Cl^- < F^-$
- Reducing power of hydrogen halides  
 $HF < HCl < HBr < HI$
- Dipole moment of hydrogen halides  
 $HF > HCl > HBr > HI$
- Oxidising power of oxides of chlorine  
 $Cl_2O > ClO_2 > Cl_2O_6 > Cl_2O_7$
- Acidic character of oxyacids of chlorine  
 $HClO < HClO_2 < HClO_3 < HClO_4$
- Strength of conjugate bases of oxyacids of chlorine  
 $ClO^- > ClO_2^- > ClO_3^- > ClO_4^-$
- Oxidising power of oxyacids of chlorine  
 $HClO > HClO_2 > HClO_3 > HClO_4$
- thermal stability of oxyacids of chlorine  
 $HClO < HClO_2 < HClO_3 < HClO_4$
- Stability of anions of oxyacids of chlorine  
 $ClO^- < ClO_2^- < ClO_3^- < ClO_4^-$

**TRANSITION ELEMENTS (D-BLOCK ELEMENTS)**

- The element with exceptional configuration are  
 $Cr^{24} [Ar] 3d^5 4s^1$ ,  $Cu^{29} [Ar] 3d^{10} 4s^1$   
 $Mo^{42} [Kr] 4d^5 5s^1$ ,  $Pd^{46} [Kr] 4d^{10} 5s^0$   
 $Ag^{47} [Kr] 4d^{10} 5s^1$ ,  $Pt^{78} [Xe] 4f^{14} 5d^{10} 6s^0$   
 $Au^{97} [Xe] 5f^{14} 5d^{10} 6s^1$
- Ferromagnetic substances are those in which there are large number of electrons with unpaired spins and whose magnetic moments are aligned in the same direction.

**ORGANIC CHEMISTRY****GOC**

- The order of decreasing electronegativity of hybrid orbitals is  $sp > sp^2 > sp^3$ .
- conformational isomers are those isomers which arise due to rotation around a single bond.
- A meso compound is optically inactive, even though it

has asymmetric centres (due to internal compensation of rotation of plane polarised light)

- An equimolar mixture of enantiomers is called racemic mixture, which is optically inactive.
- Tautomerism is the type of isomerism arising by the migration of hydrogen.
- Reaction intermediates and reagents:  
Homolytic fission  $\rightarrow$  Free radicals  
Heterolytic fission  $\rightarrow$  Carbocation and carbanion
- Nucleophiles – electron rich  
Two types : 1. Anions 2. Neutral molecules with lone pair of electrons (Lewis bases)  
Electrophiles: electron deficient.  
Two types : 1. Cations 2. Neutral molecules with vacant orbitals (Lewis acids).
- Inductive effect is due to  $\sigma$  electron displacement along a chain and is permanent effect.
- +I (inductive effect) increases basicity, –I effect increases acidity of compounds.
- Resonance is a phenomenon in which two or more structures can be written for the same compound but none of them actually exists.

**ALKANES**

- Pyrolytic cracking is a process in which alkane decomposes to a mixture of smaller hydrocarbons, when it is heated strongly, in the absence of oxygen,
- Combustion is a process in which hydrocarbons form carbon dioxide and  $H_2O$  ( $l$ ) when they are completely burnt in air/ $O_2$ .

**ALKENES**

- In dehydration and dehydrohalogenation the preferential order for removal of hydrogen is  $3^\circ > 2^\circ > 1^\circ$  (Saytzeff's rule).
- The lower the  $\Delta H_h$  (heat of hydrogenation) the more stable the alkene is.
- alkenes undergo anti-Markonikov addition only with HBr in the presence of peroxides.

**ALKYNES**

- Alkynes add water molecule in presence of mercuric sulphate and dil.  $H_2SO_4$  and form carbonyl compounds.
- Terminal alkynes have acidic H-atoms, so they form metal alkynides with Na, ammonical cuprous chloride solution and ammonical silver nitrate solution.
- Alkynes are acidic because of H-atoms which are attached to  $sp$  hybridised 'C' atom has more electronegativity as it has more 's' character than  $sp^2$  and  $sp^3$  hybridised 'C' atoms.

**ARENES**

- All o and p-directing groups are ring activating groups (except -X)  
The are : -OH, -NH<sub>2</sub>, -X, -R, -OR etc.
- All m-directing groups are ring deactivating groups.  
They are : CHO, -COOH, -NO<sub>2</sub>, -CN, -N<sup>+</sup>R<sub>3</sub> etc.

**HALOGEN COMPOUNDS**

- The order of reactivity is  
(a) RI > RBr > RCl > RE  
(b) allyl halide > Alkyl halide > vinyl halide  
(c) Alkyl halide > Aryl halide
- S<sub>N</sub>1 reaction: Mainly 3° alkyl halides undergo this reaction and form racemic mixture. S<sub>N</sub>1 is favoured by polar solvent and low concentration of nucleophile.
- S<sub>N</sub>2 reaction: Mainly 1° alkyl halides undergo this substitution. S<sub>N</sub>2 reaction is preferred by non-polar solvents and high concentration of nucleophile.

**ALCOHOLS**

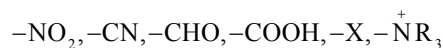
- Alkenes are converted to alcohol in different ways as follows
 

Reagent	types of addition	
dil H <sub>2</sub> SO <sub>4</sub>	- Markovnikow	
B <sub>2</sub> H <sub>6</sub> and H <sub>2</sub> O <sub>2</sub> , OH <sup>-</sup>	- Anti-Markovnikow	
Oxymercuration demercuration	-Markovnikow	
- Oxidation of
 

1° alcohol	→ aldehyde	→ carboxylic acid
	(with same no. of C atom)	(with same no. of C atom)
2° alcohol	→ ketone	→ carboxylic acid
	(with same no. of C atom)	(with less no. of C atom)
3° alcohol	→ ketone	→ carboxylic acid
	(with less no. of C atom)	(with less no. of C atom)

**PHENOLS**

- Phenol  $\xrightarrow{\text{CHCl}_3/\text{OH}^\ominus}$  Salicylaldehyde  
(Reimer-Tieman reaction)
- Phenol  $\xrightarrow[\Delta]{\text{CO}_2}$  Salicylic acid (Kolbe reaction)
- Acidity of phenols  
(a) Increases by electron withdrawing substituents like



- (b) Decrease by electron relasing substituents like  
-R, -OH, -NH<sub>2</sub>, -NR<sub>2</sub>, -OR

**ETHERS**

- $2\text{ROH} \xrightarrow[250^\circ\text{C}]{\text{Al}_2\text{O}_3} \text{R}-\text{O}-\text{R} + \text{H}_2\text{O}$
- $\text{RONa} + \text{X}-\text{R}' \longrightarrow \text{ROR}' + \text{NaAX}$   
(Williamson's synthesis)
- $\text{ROR} + 2\text{H}_2\text{SO}_4 \xrightarrow[\text{(conc)}]{\Delta} 2\text{RHSO}_4 + \text{H}_2\text{O}$
- $\text{ROR} + \text{H}_2\text{O} \xrightarrow[\Delta]{\text{dil. H}_2\text{SO}_4} 2\text{ROH}$

**CARBONYL COMPOUNDS**

- Formation of alcohols using RMgX  
(a) Formaldehyde + RMgX  $\xrightarrow{\text{Hydrolysis}}$  1° alcohol  
(b) aldehyde + RMgX  $\xrightarrow{\text{Hydrolysis}}$  2° alcohol  
(other than HCHO)  
(c) Ketone + RMgX  $\xrightarrow{\text{Hydrolysis}}$  3° alcohol
- Cannizzaro reaction (Disproportionation)  
Aldehyde  $\xrightarrow[\text{alkali}]{\text{HO}^\ominus\text{conc.}}$  Alcohol + Salt of Acid  
(no α H-atom)  
Crossed -Cannizzaro reaction gives alcohol with aryl group or bigger alkyl group.
- Aldol condensation:  
Carbonyl compound + dil. Alkali  $\longrightarrow$  β-hydroxy(carbonyl) compound  
(with α H-atom)
- Benzoin condensation  
Benzaldehyde  $\xrightarrow[\text{NaCN}]{\text{ethanolic}}$  Benzoin

**NITROGEN COMPOUNDS**

- Order of basicity : (R = -CH<sub>3</sub> or -C<sub>2</sub>H<sub>5</sub>)  
2° > 1° > 3° > NH<sub>3</sub>
- Hofmann degradation  
Amides  $\xrightarrow{\text{Br}_2/\text{KOH}}$  1° amine
- The basicity of amines is  
(a) decreased by electron withdrawing groups  
(b) increased by electron releasing groups
- Reduction of nitrobenzone in different media gives different products
 

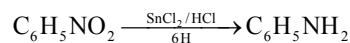
<b>Medium</b>	<b>Product</b>
Acidic	Aniline

Basic	Azoxy, Azo and finally hydrazobenzene
Neutral	Phenyl hydroxylamine
Tests to differentiate:	
1°, 2° and 3° alcohols	Lucas test Victormayer's test
1°, 2° and 3° amines	Hinsberg test
1°, 2° and 3° nitro compounds	Test with HNO <sub>2</sub> and KOH
Aryl halides and alkyl halides	Test with AgNO <sub>3</sub> Solution
Aldehydes and ketones	Tollen's Test/Fehlings test
Aromatic aldehydes and	Fehling's Test
Aliphatic aldehydes	

**IMPORTANT REAGENT**

- Dil H<sub>2</sub>SO<sub>4</sub> [or conc. H<sub>2</sub>SO<sub>4</sub> + H<sub>2</sub>O]  
Use → Dehydrating agent (+HOH)  
(a)  $\text{CH}_2 = \text{CH}_2 \xrightarrow{\text{dil. H}_2\text{SO}_4} \text{CH}_3 - \text{CH}_2 - \text{OH}$   
(b)  $\text{C}_2\text{H}_5\text{OC}_2\text{H}_5 \xrightarrow{\text{dil. H}_2\text{SO}_4} 2\text{C}_2\text{H}_5\text{OH}$
- Alc. KOH or NaNH<sub>2</sub> (Use → -HX)  
 $\text{CH}_3\text{CH}_2\text{Cl} \xrightarrow[\text{-HCl}]{\text{alc. KOH}} \text{CH}_2 = \text{CH}_2$
- Cu or ZnO/300°C  
1° alc → ald, 2° alc → ketone.  
3° alc → alkene (exception)
- Lucas reagent ZnCl<sub>2</sub> + Conc. HCl  
Use → for distinction between 1°, 2° & 3° alcohol
- Tilden Reagent NOCl (Nitrosyl chloride)  
 $\text{C}_2\text{H}_5\text{NH}_2 \xrightarrow{\text{NOCl}} \text{C}_2\text{H}_5\text{Cl}$
- Alkaline KMnO<sub>4</sub> (Strong oxidant)  
Toluene → Benzoic acid
- Bayer's Reagent  
1% alkaline KMnO<sub>4</sub> (Weak oxidant)  
Use → For test of >C=C< or -C≡C-  
 $\text{CH}_2 = \text{CH}_2 + \text{H}_2\text{O} + [\text{O}] \longrightarrow \text{CH}_2\text{OH} - \text{CH}_2\text{OH}$
- Acidic K<sub>2</sub>Cr<sub>2</sub>O<sub>7</sub> (Strong oxidant)  
 $\text{RCH}_2\text{OH} \xrightarrow{[\text{O}]} \text{RCHO}$

- SnCl<sub>2</sub>/HCl or Sn/HCl use → for reduction of nitrobenzene in acidic medium.



- Lindlar's Catalyst = Pb/CaCO<sub>3</sub>  
+ in small quantity (CH<sub>3</sub>COO)<sub>2</sub>Pb  
2-butene + H<sub>2</sub> → Cis-2butene  
(main product)
- Ziegler-Natta Catalyst (C<sub>2</sub>H<sub>5</sub>)<sub>3</sub>Al + TiCl<sub>4</sub>  
Use - Ln Addition polymerisation  
Propene → Poly propene

**MAIN USE OF COMPOUNDS**

Alkane → Fuel, alkene → polymer, Alkyne → Solvent making westron, Westrosol, General alkyl halide → as solvents, CHCl<sub>3</sub> → Anaesthetic, Germicide, CCl<sub>4</sub> → Pyrene & Fire extinguisher, CH<sub>3</sub>OH → Antifreeze, deforming of alcohol, C<sub>2</sub>H<sub>5</sub>OH → Tonic, wine preparation, Power alcohol, C<sub>2</sub>H<sub>5</sub>-O-C<sub>2</sub>H<sub>5</sub> → Antiseptic, Natellite, HCHO → Formamint medicine, CH<sub>3</sub>CHO → Antiseptic, CH<sub>3</sub>COCH<sub>3</sub> → as solvent, CH<sub>3</sub>COOC<sub>2</sub>H<sub>5</sub> → Artificial silk & flavour, CH<sub>3</sub>NH<sub>2</sub> → Refrigerating agent, C<sub>2</sub>H<sub>5</sub>NH<sub>2</sub> → in development of photography.

**IDENTIFICATION TESTS**

- Unsaturated compound (Bayer's reagent)  
Decolourising the reagent
- Alcohols (Ceric ammonium nitrate solution)  
Red colouration
- Phenols (neutral FeCl<sub>3</sub> solution)  
Violet/deep blue colouration
- Aldehydes and ketones (2, 4-D.N.P.)  
Orange precipitate
- Acids (NaHCO<sub>3</sub> solution)  
Brisk effervescence (CO<sub>2</sub> is evolved)
- 1° amine (CHCl<sub>3</sub> + KOH)  
Foul smell (isocyanide)
- 2° amine (NaNO<sub>2</sub> + HCl)  
Yellow oily liquid (Nitrosoamine)



Engineering  
IIT-JEE

Where is  
my career?

Medical  
PMT/NEET

Where are you going? Where do you want to be?  
inspiring your future



**APEX EDUCATION**

IIT-JEE / PMT / FOUNDATION / CBSE

Success through hard work & disciplined study ...

## MATHEMATICS

### RELATIONS AND FUNCTIONS

1. A function  $f: X \rightarrow Y$  is one-one (or injective) if  
 $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \quad \forall x_1, x_2 \in X$ .
2. A function  $f: X \rightarrow Y$  is onto (or surjective) if given any  
 $y \in Y, \exists x \in X$  such that  $f(x) = y$ . [codomain = range]
3. A function  $f: X \rightarrow Y$  bijective if is both one-one and onto.
4. The composition of function  $f: A \rightarrow B$  and  $g: B \rightarrow C$  is  
the function  $g \circ f: A \rightarrow C$  given by  $g \circ f(x) = g(f(x))$   
 $\forall x \in A$ .
5. A function  $f: X \rightarrow Y$  is invertible if  $\exists g: Y \rightarrow X$  such  
that  $g \circ f = I_X$  and  $f \circ g = I_Y$ .
6. A function  $f: X \rightarrow Y$  is invertible if and only if  $f$  is one-  
one and onto.

### TRIGONOMETRIC FUNCTION

1.  $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
2.  $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
3.  $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$
4.  $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$
5.  $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$
6.  $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$
7.  $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$
8.  $\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots$  ..... to  $n$  terms

$$= \frac{\sin \left[ \alpha + \left( \frac{n-1}{2} \right) \beta \right] \left[ \sin \left( \frac{n\beta}{2} \right) \right]}{\sin \left( \frac{\beta}{2} \right)} ; \beta \neq 2n\pi$$

9.  $\cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots$  to  $n$  terms

$$= \frac{\cos \left[ \alpha + \left( \frac{n-1}{2} \right) \beta \right] \left[ \sin \left( \frac{n\beta}{2} \right) \right]}{\sin \left( \frac{\beta}{2} \right)} ; \neq 2n\pi$$

10. Sine rule :  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$
11. Cosine Rule :  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ ,  
 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ ,  $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ .
12.  $\tan \left( \frac{B-C}{2} \right) = \left( \frac{b-c}{b+c} \right) \cot \left( \frac{A}{2} \right)$
13.  $\sin \left( \frac{A}{2} \right) = \sqrt{\frac{(s-b)(s-c)}{bc}}$
14.  $\tan \left( \frac{A}{2} \right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
15.  $\Delta = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B = \frac{abc}{4R} = rs$ .  $R$  =  
circum radius,  $r$  = in-radius
16.  $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$
17.  $r = 4R \sin \left( \frac{A}{2} \right) \sin \left( \frac{B}{2} \right) \sin \left( \frac{C}{2} \right)$
18.  $a = c \cos B + b \cos C$   $b = c \cos A + a \cos C$

### INVERSE TRIGONOMETRIC FUNCTIONS

1. Domain and range of inverse trigonometric function

Function	Domain	Range
$y = \sin^{-1}x$	$[-1, 1]$	$\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$

$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	$(-\infty, \infty)$	$[0, \pi]$
$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
$y = \operatorname{cosec}^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

2. Some properties of inverse trigonometric function:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}; \quad x \in [-1, 1]$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}; \quad x \in \mathbb{R}$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2} \quad x \in (-\infty, -1] \cup [1, \infty)$$

$$2 \sin^{-1} x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}} \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } \frac{1}{\sqrt{2}} \leq x \leq 1 \\ -\pi - \sin^{-1}(2x\sqrt{1-x^2}), & \text{if } -1 \leq x \leq -\frac{1}{\sqrt{2}} \end{cases}$$

$$2 \tan^{-1} x = \begin{cases} \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } -1 < x < 1 \\ \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } x > 1 \\ -\pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), & \text{if } x < -1 \end{cases}$$

**QUADRATIC EQUATIONS AND INEQUALITIES**

1. For the quadratic equation  $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \alpha + \beta = \frac{-b}{a} \quad \alpha\beta = \frac{c}{a}$$

(i) If  $b = 0 \Rightarrow$  roots are of equal magnitude but of opposite sign

(ii) If  $a = c \Rightarrow$  roots are reciprocal to each other

(iii)  $\left. \begin{matrix} a > 0, & c < 0 \\ a < 0, & c > 0 \end{matrix} \right\} \Rightarrow$  Roots are of opposite signs.

(iv) If  $\left. \begin{matrix} a > 0, b > 0, c > 0 \\ a < 0, b < 0, c < 0 \end{matrix} \right\} \Rightarrow$  both roots are negative

(vii) If  $\left. \begin{matrix} a > 0, b < 0, c > 0 \\ a < 0, b > 0, c < 0 \end{matrix} \right\} \Rightarrow$  both roots are positive

(viii) If sign of  $a =$  sign of  $b \neq$  sign of  $c \Rightarrow$  Greater root in magnitude is negative

- If  $p + iq$  ( $p$  and  $q$  being real) is a root of the quadratic equation, where  $i = \sqrt{-1}$ , then  $p - iq$  is also a root of the quadratic equation.
- Every equation of  $n^{\text{th}}$  degree ( $n \geq 1$ ) has exactly  $n$  roots and if the equation has more than  $n$  roots, it is an identity.
- An inequality of the form  $\log_a f(x) > b$  is equivalent to the following systems of inequalities:  
(a)  $f(x) > 0, f(x) > a^b$  for  $a > 1$   
(b)  $f(x) > 0, f(x) < a^b$  for  $a < 1, a > 0$
- Irrational roots occurs in pairs  $(p + \sqrt{q})$  and  $(p - \sqrt{q})$

**COMPLEX NUMBERS**

- For any integer  $k, i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i$
- $|z - z_1| + |z - z_2| = \lambda$ , represents an ellipse if  $|z_1 - z_2| < \lambda$ , having the points  $z_1$  and  $z_2$  as its foci. And if  $|z_1 - z_2| = \lambda$ , then  $z$  lies on a line segment connecting  $z_1$  and  $z_2$
- Cube roots of unity  $(1, \omega, \omega^2)$   
(i)  $\omega^3 = 1$                       (ii)  $1 + \omega + \omega^2 = 0$

The three cube roots of unity are

$$1, \omega = \frac{1}{2}(-1 + \sqrt{3}i), \omega^2 = \frac{1}{2}(-1 - \sqrt{3}i) \text{ which are the same as}$$

$$1, \cos\left(\frac{2\pi}{3}\right) + i \sin\left(\frac{2\pi}{3}\right) \text{ and } \cos\left(\frac{4\pi}{3}\right) + i \sin\left(\frac{4\pi}{3}\right)$$

4. De Moivre's theorem: For all real values of  $n$ ,

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

If  $z = \cos \theta + i \sin \theta$  using De-Moivre's theorem

$$z^n + \frac{1}{z^n} = 2 \cos n\theta; \quad z^n - \frac{1}{z^n} = 2i \sin n\theta$$

**PERMUTATIONS AND COMBINATIONS**

- The number of permutations of  $n$  different things taken  $r$  at a time, where repetition is not allowed, is denoted by  ${}^n P_r$  and is given by  ${}^n P_r = \frac{n!}{(n-r)!}$ , where  $0 \leq r \leq n$
- The number of permutations of  $n$  different things, taken  $r$  at a time, where repetition is allowed, is  $n^r$ .

3. The number of permutations of  $n$  objects taken all at a time, where  $p_1$  objects are of first kind,  $p_2$  objects are of the second kind, ...  $p_k$  objects are of the  $k^{\text{th}}$  kind and rest, if

any are all different is  $\frac{n!}{p_1!p_2!\dots p_k!}$

4. The number of combination of  $n$  different things taken  $r$  at a time, denoted by  ${}^nC_r$ , is given by

$${}^nC_r = \frac{n!}{r!(n-r)!}, 0 \leq r \leq n$$

5. Number of circular permutations of  $n$  things when  $p$  alike and the rest different taken all at a time distinguish

clockwise and anticlockwise arrangement is  $\frac{(n-1)!}{p!}$

### BINOMIAL THEOREM

1. The expansion of a binomial for any positive integral  $n$  is given by Binomial Theorem, which is

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n$$

2. The general term of an expansion  $(a+b)^n$  is

$$T_{r+1} = {}^nC_r a^{n-r} b^r.$$

The total number of terms in the expansion of  $(a+b)^n$  is  $n+1$ .

3. In the expansion  $(a+b)^n$ , if  $n$  is even, then the middle term

is the  $\left(\frac{n}{2}+1\right)^{\text{th}}$  term. If  $n$  is odd, then the middle terms are

$$\left(\frac{n+1}{2}\right)^{\text{th}} \text{ and } \left(\frac{n+3}{2}\right)^{\text{th}} \text{ terms.}$$

### SEQUENCE AND SERIES

1. The general term or the  $n^{\text{th}}$  term of the A.P. is given by  $a_n = a + (n-1)d$ .

The sum  $S_n$  of the first  $n$  terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + \ell)$$

2. The sum  $S_n$  of the first  $n$  terms of G.P. is given by

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ or } \frac{a(1 - r^n)}{1 - r}, \text{ if } r \neq 1, S_\infty = \frac{a}{1 - r}$$

3. A series whose each term is formed, by multiplying corresponding terms of an A.P. and a G.P. is called an Arithmetic-geometric series. Summation of  $n$  terms: \

$$S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a + (n-1)d]r^n}{1-r}$$

4. Harmonical progression is defined as a series in which reciprocal of its terms are in A.P.

The standard form of a H.P. is  $\frac{1}{a} + \frac{1}{a+d} + \frac{1}{a+2d} + \dots$

### STRAIGHT LINES

1. An acute angle (say  $\theta$ ) between lines  $L_1$  and  $L_2$  with slopes

$m_1$  and  $m_2$  is given by  $\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|, 1 + m_1 m_2 \neq 0$

2. The perpendicular distance ( $d$ ) of a line  $Ax + By + C = 0$

from a point  $(x_1, y_1)$  is given by  $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$ .

3. Distance between the parallel lines  $Ax + By + C_1 = 0$  and

$Ax + By + C_2 = 0$ , is given by  $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$

### CONIC SECTIONS

1. The equation of a circle with centre  $(h, k)$  and the radius  $r$  is  $(x-h)^2 + (y-k)^2 = r^2$ .

2. Equation of tangent :  $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

3. The equation of the parabola with focus at  $(a, 0)$   $a > 0$  directrix  $x = -a$  is  $y^2 = 4ax$ .

4. Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola.

5. Length of the latus rectum of the parabola  $y^2 = 4ax$  is  $4a$ .

6. The parametric equation of the parabola is  $x = at^2, y = 2at$

7. An ellipse is the set of all points in a plane, the sum of whose distance from two fixed points in the plane is a constant.

8. The equations of an ellipse with foci on the  $x$ -axis is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \text{ its parametric equation is } x = a \cos \theta; y = b \sin \theta$$

9. Latus rectum of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is  $\frac{2b^2}{a}$

10. The eccentricity of an ellipse is the ratio between the distances from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse.

11. A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

12. The equation of hyperbola with foci on the  $x$ -axis is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1; \text{ Two asymptotes: } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

13. Letus rectum of the hyperbola :  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is  $\frac{2b^2}{a}$

### THREE DIMENSIONAL GEOMETRY

1. The coordinates of the point R which divides the line segment joining two points P ( $x_1, y_1, z_1$ ) and Q( $x_2, y_2, z_2$ ) internally and externally in the ratio  $m : n$  are given by

$$\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$$

and  $\left( \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$ , respectively.

2. The coordinates of the centroid of the triangle, whose vertices are ( $x_1, y_1, z_1$ ), ( $x_2, y_2, z_2$ ) and ( $x_3, y_3, z_3$ ) are

$$\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

3. If  $\ell, m, n$  are the direction cosines and  $a, b, c$  are the direction ratios of a line then

$$\ell = \frac{\pm a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{\pm b}{\sqrt{a^2 + b^2 + c^2}}, n = \frac{\pm c}{\sqrt{a^2 + b^2 + c^2}}$$

4. If  $\ell_1, m_1, n_1$  and  $\ell_2, m_2, n_2$  are the direction cosines of two lines; and  $\theta$  is the acute angle between the two lines; then  $\cos \theta = |\ell_1 \ell_2 + m_1 m_2 + n_1 n_2|$

5. Equation of a line through a point ( $x_1, y_1, z_1$ ) and having direction cosines  $\ell, m, n$  is  $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

6. Shortest distance between  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$

$$\text{is } \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

7. The equation of a plane through a point whose position vector is  $\vec{a}$  and perpendicular to the vector  $\vec{N}$  is

$$(\vec{r} - \vec{a}) \cdot \vec{N} = 0$$

8. Vector equation of a plane that passes through the line of intersection of planes  $\vec{r} \cdot \vec{n}_1 = d_1$  and  $\vec{r} \cdot \vec{n}_2 = d_2$  is  $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$ , where  $\lambda$  is any nonzero constant.

9. The distance of a point whose position vector is  $\vec{a}$  from the plane  $\vec{r} \cdot \vec{n} = d$  is  $|d - \vec{a} \cdot \vec{n}|$ .

10. Skew line: Two straight lines are said to be skew lines if

they are neither parallel nor intersecting.

$$\text{Shortest distance: } \frac{\Sigma(x_2 - x_1)(m_1 n_2 - m_2 n_1)}{\sqrt{\Sigma(m_1 n_2 - m_2 n_1)^2}}$$

### DIFFERENTIAL CALCULUS

#### LIMIT

1. A function  $f$  has the limit  $L$  as  $X$  approaches  $a$  if the limit from the left exists and the limit from the right exists and both limit

$$\text{are } L \text{ that is, } \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

Remember  $\lim_{x \rightarrow a} x = a$

Let  $\lim_{x \rightarrow a} f(x) = \ell$  and  $\lim_{x \rightarrow a} g(x) = m$ . If  $\ell$  and  $m$  are finite then:

(i)  $\lim_{x \rightarrow a} (f(x) \pm g(x)) = \ell \pm m$

(ii)  $\lim_{x \rightarrow a} f(x) \cdot g(x) = \ell \cdot m$

(iii)  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\ell}{m}$ , Provided  $m \neq 0$

(iv)  $\lim_{x \rightarrow a} k f(x) = k \lim_{x \rightarrow a} f(x) = k\ell$ ; where  $k$  is a constant.

(v)  $\lim_{x \rightarrow a} [f(x) + k] = \lim_{x \rightarrow a} f(x) + k$ , where  $k$  is a constant.

(vi) If  $f(x) \leq g(x)$ , then  $\lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} g(x)$ .

2. Limit in case of Composite Function:

$$\lim_{x \rightarrow a} f[g(x)] = f\left(\lim_{x \rightarrow a} g(x)\right); \text{ provided } f \text{ is continuous at } x.$$

3.  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = \lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = \lim_{x \rightarrow 0} \frac{x}{\sin^{-1} x} = 1$

(where  $x$  is measured in radian)

4.  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow 0} (1+x)^{1/x} = e$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = \lim_{x \rightarrow 0} (1+x)^{a/x} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{ax} = e^a$$

#### CONTINUITY OF FUNCTIONS

- All polynomials, Trigonometrical, exponential and Logarithmic functions are continuous in their domain.
- If  $f(x)$  is continuous and  $g(x)$  is discontinuous at  $x = a$  then the product function  $\phi(x) = f(x)g(x)$  is not necessarily be

discontinuous at  $x = a$ . e.g.  $f(x)$  &  $g(x) = \begin{cases} \sin \frac{\pi}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

- For any positive integer  $n$  and any continuous function  $f$ ,  $[f(x)]^n$  and  $\sqrt[n]{f(x)}$  are continuous when  $n$  is even the input of  $f$  in  $\sqrt[n]{f(x)}$  restricted to inputs  $x$  for which  $f(x) \geq 0$ .
- If  $f(x)$  and  $g(x)$  are continuous, then so are  $f(x) + g(x)$ ,  $f(x) - g(x)$ , and  $f(x) \cdot g(x)$
- If  $f(x)$  and  $g(x)$  are continuous, so is  $g(x)/f(x)$ , so long as the inputs  $x$  do not yield outputs  $f(x) = 0$ .

**DIFFERENTIATION AND APPLICATION**

- Interpretation of the Derivative:** If  $y = f(x)$  then,
  - $m = f'(a)$  is the slope of the tangent line to  $y = f(x)$  at  $x = a$  and the equation of the tangent line at  $x = a$  is given by  $y = f(a) + f'(a)(x - a)$ .
  - $f'(a)$  is the instantaneous rate of changes of  $f(x)$  at  $x = a$ .
  - If  $f(x)$  is the position of an object at time  $x$  then  $f'(a)$  is the velocity of the object at  $x = a$ .
- Basic Properties and Formulas :** If  $f(x)$  and  $g(x)$  are differentiable function  $s$  (the derivative exists),  $c$  and  $n$  are any real numbers

1.  $(cf)' = cf'(x)$                       2.  $(f \pm g)' = f'(x) \pm g'(x)$

3.  $(fg)' = f'g + fg'$                       4.  $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

5.  $\frac{d}{dx}(x^n) = nx^{n-1}$

6.  $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

**3. Increasing/Decreasing:**

- If  $f'(x) > 0$  for all  $x$  in an interval  $I$  then  $f(x)$  is increasing on the interval  $I$ .
- If  $f'(x) < 0$  for all  $x$  in an interval  $I$  then  $f(x)$  is decreasing on the interval  $I$ .
- If  $f'(x) = 0$  for all  $x$  in an interval  $I$  then  $f(x)$  is constant on the interval  $I$ .

**4. Concave Up/Concave Down:**

- If  $f''(x) > 0$  for all  $x$  in an interval  $I$  then  $f(x)$  is concave up on the interval  $I$ .
- If  $f''(x) < 0$  for all  $x$  in an interval  $I$  then  $f(x)$  is concave down on the interval  $I$ .

**5. 1<sup>st</sup> Derivative Test:** If  $x = c$  is a critical point of  $f(x)$  then  $x = c$  is

- a rel. max. of  $f(x)$  if  $f'(x) > 0$  to the left of  $x = c$  and

$f'(x) < 0$  to the right of  $x = c$ .

- a rel. min. of  $f(x)$  if  $f'(x) < 0$  to the left of  $x = c$  and  $f'(x) > 0$  to the right of  $x = c$ .
- not a relative extrema of  $f(x)$  if  $f'(x)$  is the same sign on both sides of  $x = c$ .

2nd Derivative Test: If  $x = c$  is critical point of  $f(x)$  such that  $f'(c) = 0$  then  $x = c$

- is a relative maximum of  $f(x)$  if  $f''(c) < 0$
- is a relative minimum of  $f(x)$  if  $f''(c) > 0$
- $f(x)$  may have a relative maximum, relative minimum, or neither if  $f''(c) = 0$

**6. Mean value theorem:** If  $f(x)$  is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$

then there is a number  $a < c < b$  such that  $f'(c) = \frac{f(b) - f(a)}{b - a}$

**7. Rolle's theorem:** If a function  $f(x)$  is continuous on the closed interval  $[a, b]$  and differentiable in an interval  $(a, b)$  and also  $f(a) = f(b)$ , then there exist at least one value  $c$  of  $x$  in the interval  $(a, b)$  such that  $f'(c) = 0$ .

**8. L'Hospital's rule:**  $\lim_{x \rightarrow a} \frac{f(x)}{\phi(x)} = \frac{f'(a)}{\phi'(a)} = \frac{f''(a)}{\phi''(a)}$

if  $f(a) = 0$  or  $\infty$ ,  $\phi(a) = 0$  or  $\infty$ ,  $f'(a) = 0$  or  $\infty$ ,  $\phi'(a) = 0$  or  $\infty$

**9. Length of sub-tangent** =  $\left| y_1 \left( \frac{dx}{dy} \right)_{(x_1, y_1)} \right|$ ; Sub-normal

=  $\left| y_1 \left( \frac{dy}{dx} \right)_{(x_1, y_1)} \right|$

Length of tangent =  $\left| y_1 \sqrt{1 + \left( \frac{dx}{dy} \right)_{(x_1, y_1)}^2} \right|$

Length of normal =  $\left| y_1 \left\{ 1 + \left( \frac{dy}{dx} \right)_{(x_1, y_1)}^2 \right\} \right|$

**10. Orthogonal trajectory:** Any curve which cuts every member of a given family of curves at right angle, is called an orthogonal trajectory of the family.

**INTEGRAL CALCULUS**

**1. Some properties of definite integral:**

- If an interval  $[a, b]$  ( $a < b$ ), the function  $f(x)$  and  $\phi(x)$  satisfy the condition  $f(x) \leq \phi(x)$ , then

$$\int_b^a f(x) dx \leq \int_b^a \phi(x) dx$$

- (b) If  $m$  and  $M$  are the smallest and greatest values of a function  $f(x)$  on an interval  $[a, b]$  and  $a \leq b$ , then

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$$

(c)  $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ , where  $a < c < b$

(d)  $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

(e)  $\int_0^{na} f(x)dx = n \int_0^a f(x)dx$  where  $a$  is the period of the function and  $n \in I$

(f)  $\int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, & \text{If } f(x) \text{ is even function i.e. } f(-x) = f(x) \\ 0, & \text{If } f(x) \text{ is odd function, i.e. } f(x) = -f(-x) \end{cases}$

(g)  $\int_0^{2a} f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, & \text{If } f(2a-x) = f(x) \\ 0, & \text{If } f(2a-x) = -f(x) \end{cases}$

2. Leibnitz rule :

$$\frac{d}{dx} \int_{f(x)}^{g(x)} F(t) dt = g'(x)F(g(x)) - f'(x)F(f(x))$$

3. If series can be put in the form

$$\frac{1}{n} \sum_{r=0}^{r=n-1} f\left(\frac{r}{n}\right) \text{ or } \frac{1}{n} \sum_{r=1}^{r=n} f\left(\frac{r}{n}\right), \text{ then its limits as}$$

$$n \rightarrow \infty \text{ is } \int_0^1 f(x) dx$$

### PROBABILITY

1. Probability of an event : For a finite sample space with equally likely outcomes Probability of an event

$$P(A) = \frac{n(A)}{n(S)}, \text{ where } n(A) = \text{number of elements in the set } A, n(S) = \text{number of elements in the set } S.$$

2.  $A$  and  $B$  are any two events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

equivalently,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3. If  $A$  and  $B$  are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B).$$

4. If  $A$  is any event, the  $P(\text{not } A) = 1 - P(A)$

5. The conditional probability of an event  $E$ , given the occurrence of the event  $F$  is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

6. Theorem of total probability: Let  $\{E_1, E_2, \dots, E_n\}$  be a partition of a sample space and suppose that each of  $E_1, E_2, \dots, E_n$  has nonzero probability. Let  $A$  be any event associated with  $S$ , then  $P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) + \dots + P(E_n)P(A|E_n)$

7. **Bayes' theorem:** If  $E_1, E_2, \dots, E_n$  are events which constitute a partition of sample space  $S$ , i.e.,  $E_1, E_2, \dots, E_n$  are pairwise disjoint and  $E_1 \cup E_2 \cup \dots \cup E_n = S$  and  $A$  be any event with nonzero probability, then

$$P(E_1 | A) = \frac{P(E_1)P(A | E_1)}{\sum_{j=1}^n P(E_j)P(A | E_j)}$$

8. Let  $X$  be a random variable whose possible values  $x_1, x_2, x_3, \dots, x_n$  occur with probabilities  $P_1, P_2, P_3, \dots, P_n$  respectively.

The mean of  $X$ , denoted by  $\mu$ , is the number  $\sum_{i=1}^n x_i P_i$

The mean of a random variable  $X$  is also called the expectation of  $X$ , denoted by  $E(X)$ .

9. Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

(a) There should be a finite number of trials, (b) The trials should be independent. (c) Each trial has exactly two outcomes: success or failure. (d) The probability of success remains the same in each trial. For Binomial distribution  $B(n, p)$ ,  $P(X = x) = {}^n C_x q^{n-x} p^x$ ,  $x = 0, 1, \dots, n$  ( $q = 1 - p$ )

### MATRICES

1. Positive Integral Powers of a Matrix:

For any positive integers  $m, n$

(i)  $A^m A^n = A^{m+n}$

(ii)  $(A^m)^n = A^{mn} = (A^n)^m$

(iii)  $I^n = I^m = I$

(iv)  $A^0 = I_n$  where  $A$  is a square matrices of order  $n$ .

2. **Transpose of a matrix:** The matrix obtained from a given matrix  $A$  by changing its rows into columns or columns into rows is called transpose of matrix  $A$  and is denoted by  $A^T$  or  $A'$ . From the definition it is obvious that if order of  $A$  is  $m \times n$ , then order of  $A^T$  is  $n \times m$ .

Properties of Transpose

(i)  $(A^T)^T = A$

(ii)  $(A \pm B)^T = A^T \pm B^T$

(iii)  $(AB)^T = B^T A^T$

(iv)  $(kA)^T = k(A)^T$   $k$  is scalar

(v)  $I^T = I$

(vi)  $\text{tr}(A) = \text{tr}(A)^T$

(vii)  $(A_1 A_2 A_3 \dots A_{n-1} A_n)^T = A_n^T A_{n-1}^T \dots A_3^T A_2^T A_1^T$

**3. Inverse of a matrix :** If A and B are two matrices such that  $AB = I = BA$  the B is called the inverse of A and it is denoted by  $A^{-1}$ , thus  $A^{-1} = B \Leftrightarrow AB = I = BA$

To find inverse matrix of a given matrix A we use following

formula  $A^{-1} = \frac{\text{adj.}A}{|A|}$

Thus  $A^{-1}$  exists  $\Leftrightarrow |A| \neq 0$

**DETERMINANTS**

**1. Minor :** The determinant that is left by cancelling the row and column intersecting at a particular element is called the minor of that element.

**2. Cofactor:** The cofactor of an element  $a_{ij}$  is denoted by  $F_{ij}$  and is equal to  $(-1)^{i+j} M_{ij}$  where M is a minor of element  $a_{ij}$

**3. Multiplication of two diterminants:** Multiplication of two third order determinants is defined as follows:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} =$$

$$\begin{vmatrix} a_1 l_2 + b_1 c_1 + c_1 l_3 & a_1 m_1 + b_1 m_2 + c_1 m_3 & a_1 n_1 + b_1 n_2 + c_1 n_3 \\ a_2 l_1 + b_2 l_2 + c_2 l_3 & a_2 m_1 + b_2 m_2 + c_2 m_3 & a_2 n_1 + b_2 n_2 + c_2 n_3 \\ a_3 l_1 + b_3 l_2 + c_3 l_3 & a_3 m_1 + b_3 m_2 + c_3 m_3 & a_3 n_1 + b_3 n_2 + c_3 n_3 \end{vmatrix}$$

**4. System of linear equation in three unknowns:** Using Crammer’s rule of determinant we get

$$\frac{x}{\Delta_1} = \frac{y}{\Delta_2} = \frac{z}{\Delta_3} = \frac{1}{\Delta} \text{ i.e. } x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$$

**Case - I :** If  $\Delta \neq 0$

Then  $x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$

∴ The system is consistent and has unique solution.

**Case - II** if  $\Delta = 0$  and

(i) If a least one of  $\Delta_1, \Delta_2, \Delta_3$  is not zero then the system of equations is inconsistent i.e. has no solution.

(ii) If  $d_1 = d_2 = d_3 = 0$  or  $\Delta_1, \Delta_2, \Delta_3$  are all zero then the system is consistent and has infinitely many solutions.

**VECTORS**

**1. Triangle law of vector addition of two vector**

Magnitude of  $\vec{R}$ :  $\vec{R} = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

**2. Scalar Product :**  $\vec{A} \cdot \vec{B} = AB \cos \theta$  (here  $\theta$  is the angle between the vector)

**3. Vector product :**  $\vec{C} = \vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

**4. Given vectors**  $x_1 \vec{a} + y_1 \vec{b} + z_1 \vec{c}, x_2 \vec{a} + y_2 \vec{b} + z_2 \vec{c}, x_3 \vec{a} + y_3 \vec{b} + z_3 \vec{c}$  where  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar vectors,

will be coplanar if and only if  $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$

**5. Scalar triple product:**

(a) If  $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}, \vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$  and  $\vec{c} = c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k}$  then

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = [\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(b)  $[a \ b \ c]$  = volume of the parallelopiped whose coterminous edges are formed by  $\vec{a}, \vec{b}, \vec{c}$

(c)  $\vec{a}, \vec{b}, \vec{c}$  are coplanar if and only if  $[\vec{a}, \vec{b}, \vec{c}] = 0$

(d) Four points A, B, C, D with position vectors  $\vec{a}, \vec{b}, \vec{c}, \vec{d}$  respectively are coplanar if and only if  $[\vec{AB} \ \vec{AC} \ \vec{AD}] = 0$  i.e. if and only if

$$[\vec{b} - \vec{a} \ \vec{c} - \vec{a} \ \vec{d} - \vec{a}] = 0$$

(e) Volume of a tetrahedron with three coterminous edges

$$\vec{a}, \vec{b}, \vec{c} = \frac{1}{6} |[\vec{a} \ \vec{b} \ \vec{c}]|$$

(f) Volume of prism on a triangular base with three coterminous edges  $\vec{a}, \vec{b}, \vec{c} = \frac{1}{2} |[\vec{a} \ \vec{b} \ \vec{c}]|$

**6. Lagrange’s identity:**

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix} = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

**7. Reciprocal system of vectors:**

If  $\vec{a}, \vec{b}, \vec{c}$  be any three non coplanar vectors so that  $[\vec{a} \ \vec{b} \ \vec{c}] \neq 0$  then the three vectors  $\vec{a}', \vec{b}', \vec{c}'$  defined by the

equations  $\vec{a}' = \frac{\vec{b} \times \vec{c}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \vec{b}' = \frac{\vec{c} \times \vec{a}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \vec{c}' = \frac{\vec{a} \times \vec{b}}{[\vec{a} \ \vec{b} \ \vec{c}]}$  are called

the reciprocal system of vectors to the given vectors  $\vec{a}, \vec{b}, \vec{c}$ .

**8. Application of vector in geometry:**

(a) Vector equation of a straight line passing through two points  $\vec{a}$  and  $\vec{b}$  is  $\vec{r} = \vec{a} + t(\vec{b} - \vec{a})$

(b) Vector equation of a plane passing through the point  $\vec{a}, \vec{b}, \vec{c}$  is  $\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}$

or  $\vec{r} \cdot (\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}) = [\vec{a} \vec{b} \vec{c}]$

(c) Vector equation of a plane passing through the point  $\vec{a}$  and perpendicular to  $\vec{n}$  is  $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

(d) Perpendicular distance of a point P(r) from a line passing through  $\vec{a}$  and parallel to  $\vec{b}$  is given by

$$PM = \frac{|(\vec{r} - \vec{a}) \times \vec{b}|}{|\vec{b}|} = \left[ (\vec{r} - \vec{a})^2 - \left\{ \frac{(\vec{r} - \vec{a}) \cdot \vec{b}}{|\vec{b}|} \right\}^2 \right]^{1/2}$$

(e) Perpendicular distance of a point P(r) from a plane passing through  $\vec{a}$  and parallel to  $\vec{b}$  and  $\vec{c}$  is given by

$$PM = \frac{(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|}$$

**STATISTICS**

**1. Mean deviation for ungrouped data**

$$M.D.(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}, M.D.(M) = \frac{\sum |x_i - M|}{n}$$

**2. Mean deviation for grouped data**

$$M.D.(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$M.D.(M) = \frac{\sum f_i |x_i - M|}{N}, \text{ where } N = \sum f_i$$

**3. Variance and standard deviation for ungrouped data**

$$\sigma^2 = \frac{1}{n} \sum (x_i - \bar{x})^2, \sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$$

**4. Variance and standard deviation of a discrete frequency distribution**

$$\sigma^2 = \frac{1}{N} \sum f_i (x_i - \bar{x})^2, \sigma = \sqrt{\frac{1}{N} \sum f_i (x_i - \bar{x})^2}$$

**5. Variance and standard deviation of a continuous frequency distribution**

$$\sigma^2 = \frac{1}{N} \sum f_i (x_i - \bar{x})^2, \sigma = \sqrt{\frac{1}{N} \sum f_i x_i^2 - \left( \frac{\sum f_i x_i}{N} \right)^2}$$

**6. Coefficient of variation (C.V.) =  $\frac{\sigma}{\bar{x}} \times 100, \bar{x} \neq 0$**

For series with equal means, the series with lesser standard deviation is more consistent or less scattered. Q function of x or constant

APEX EDUCATION